Comparison of Robust Estimators' Performance for Detecting Outliers in Multivariate Data

Sharifah Sakinah Syed Abd Mutalib[1*], Siti Zanariah Satari[2] & Wan Nur Syahidah Wan Yusoff[3]

[1,2,3] Centre for Mathematical Sciences, College of Computing & Applied Sciences, Universiti Malaysia Pahang, Lebuhraya Tun Razak, 26300 Gambang, Pahang

[1]Faculty of Computer, Media and Technology Management, TATI University College, Jalan Panchur, Telok Kalong, 24000 Kemaman, Terengganu, Malaysia.

**Corresponding author: sharifahsakinah84@gmail.com*

Abstract

In multivariate data, outliers are difficult to detect especially when the dimension of the data increase. Mahalanobis distance (MD) has been one of the classical methods to detect outliers for multivariate data. However, the classical mean and covariance matrix in MD suffered from masking and swamping effects if the data contain outliers. Due to this problem, many studies used a robust estimator instead of the classical estimator of mean and covariance matrix. In this study, the performance of five robust estimators namely Fast Minimum Covariance Determinant (FMCD), Minimum Vector Variance (MVV), Covariance Matrix Equality (CME), Index Set Equality (ISE), and Test on Covariance (TOC) are investigated and compared. FMCD has been widely used and is known as among the best robust estimator. However, there are certain conditions that FMCD still lacks. MVV, CME, ISE and TOC are innovative of FMCD. These four robust estimators improve the last step of the FMCD algorithm. Hence, the objective of this study is to observe the performance of these five estimator to detect outliers in multivariate data particularly TOC as TOC is the latest robust estimator. Simulation studies are conducted for two outlier scenarios with various conditions. There are three performance measures, which are *pout, pmask* and *pswamp* used to measure the performance of the robust estimators. It is found that the TOC gives better performance in *pswamp* for most conditions. TOC gives better results for *pout* and *pmask* for certain conditions.

Keywords: Mahalanobis distance, Multivariate data, Outliers, Robust Estimators, Test on Covariance

1. Introduction

Outliers are abnormal data or a minority of observations that differ from the majority of the observations in the dataset (Hadi et al., 2009; Möller et al., 2005; Su & Tsai, 2011). Outlier detection for multivariate data is more difficult compared to univariate data (Hadi et al., 2009). Outliers in univariate data can easily be detected graphically through a simple plot of the data, such as box plot, scatterplot, stem-andleaf plot, and Q-Q plot (Werner, 2003; Möller et al., 2005; Su & Tsai, 2011). However, visual inspection for multivariate data does not work well and the difficulty increases when the dimension, p increase (Rousseeuw & Van Driessen, 1999; Werner, 2003; Møller et al., 2005; Herwindiati, et al., 2007; Hadi et al., 2009;).

Mahalanobis distance (MD) has been a classical method to identify outliers in the multivariate data. Mahalanobis distance is given in Eq. 1.

$$d_i(\bar{\mathbf{x}},\mathbf{S})=\sqrt{(\mathbf{x}_i-\bar{\mathbf{x}})'\mathbf{S}^{-1}(\mathbf{x}_i-\bar{\mathbf{x}})},\quad i=1,2,....,n \qquad (1)$$

However, the classical mean and covariance matrix in MD suffer from masking and swamping effects if the data contain outliers (Santos-pereira & Pires, 2013). Masking occurs when some of the outliers are left unidentified (false negative) and swamping occurs when inliers data are mistakenly identified as outliers (false positive) (Filzmoser & Todorov, 2013). To overcome this problem, many studies replace the classical estimators with robust estimators as these estimators are resistant toward outliers and named as robust MD (Herwindiati et al., 2007; P. J. Rousseeuw, 1984; P. Rousseeuw & Yohai, 1984; Peter Rousseeuw, 1985).

Fast Minimum Covariance Determinant (FMCD) proposed by Rousseeuw and Van Driessen (1999)is one of the widely used robust estimators. The basic idea of this algorithm is the Concentration step (C-step) which is the last step in the algorithm. FMCD algorithm had been proved computationally efficient. However, FMCD still needs a lot of calculation and time consuming as it used covariance determinant in the last step of the algorithm (Lim & Midi, 2016). Due to this problem, Herwindiati et al. (2007) proposed to use vector variance (VV) in the last step of this algorithm and named the new robust estimator as Minimum Vector Variance (MVV). Although MVV is much faster than FMCD, the computation time still become the problems when the dimension, $p$increase (Salleh & Djauhari, 2011). In 2013, Rohayu developed two new robust estimators which are Covariance Matrix Equality (CME) and Index Set Equality (ISE). ISE has been proven to work excellently in terms of computation time (Lim & Midi, 2016). However, according to Rohayu (2013), this problem is still open for future works. Hence, Abd Mutalib et al. (2019) proposed a new robust estimator based on the idea of CME and ISE which is named Test on Covariance (TOC).

Abd Mutalib et al. (2019) had found that TOC is an applicable and promising approach to detect outliers for multivariate data. However, Abd Mutalib et al. (2019) only provided findings for one sample size, one number of variables, and two distances of outliers. Hence, in this study further investigation is conducted for five robust estimators namely Fast Minimum Covariance Determinant (FMCD), Minimum Vector Variance (MVV), Covariance Matrix Equality (CME), Index Set Equality (ISE), and Test on Covariance (TOC) with various sample sizes, the number of variables, percentage of outliers and two different outlier scenarios. In particular, the objective of this study is to observe the applicability of the latest robust estimator, TOC in detecting outliers for various conditions.

2. Previous study

Various robust estimators such as S, M, Method of Moments (MM), Minimum Volume Ellipsoid (MVE), Minimum Covariance Determinant (MCD) and Fast-MCD (FMCD) estimators have been presented in the previous studies. Measures of performance are needed to compare the efficiency of different robust estimators (Møller et al., 2005). High breakdown point, bounded influence function and affine equivariant are the properties that a robust estimator should have (Werner, 2003; Møller et al., 2005). The breakdown value is the smallest percentage of contamination that can have an arbitrarily large effect on the estimator (Hubert & Debruyn, 2010). For practical applications, a breakdown point of 20% is usually satisfactory (Hubert & Debruyn, 2010). Bounded influence function which describe a small contamination at any point can only have a small effect on the estimator (Hubert & Debruyn, 2010). While, affine equivariant means that the estimators behave properly under affine transformations of the data (Hubert & Debruyn, 2010). In addition, efficiency and computational speed of the algorithm to compute robust estimators are required (Werner, 2003; Hubert & Debruyn, 2010). More details about the properties are available in Rousseeuw & Van Driessen (1999), Werner (2003) and Hubert & Debruyne (2010).

Rousseeuw (1985) studied whether it is possible to combine a high breakdown point estimator with an affine equivariant estimator. Three affine equivariant estimators with a high breakdown point of 50% had been discussed. The three estimators are outlyingness-weighted mean, Minimal Volume Ellipsoid (MVE) and Minimal Covariance Determinant (MCD). Outlyingness-weighted mean is an estimator obtained and related to the projection pursuit method because the best projection needs to be searched over all possible directions (Peter Rousseeuw, 1985). MVE is found to have a slow rate of convergence but the number of arithmetic operations required to compute MVE is much faster than MCD (Peter Rousseeuw, 1985). In terms of efficiency, Rousseeuw (1985) found that both MVE and MCD have low asymptotic efficiency. Additionally, both MVE and MCD are too difficult to compute precisely in moderate and large data sets (Werner, 2003).

According to Rousseeuw and Van Driessen (1999), there are several reasons to choose MCD over MVE. MCD has better statistical efficiency because MCD is asymptotically normal, robust distance based on MCD is more precise than MVE and easy to compute for small data sets. Despite the advantages of MCD over MVE, the computation of MCD is cumbersome (P. J. Rousseeuw & Van Driessen, 1999). Hence in 1999, Rousseeuw and Van Driessen developed a new algorithm for MCD which called Fast-MCD (FMCD) that is based on concentration step, C-step (Herwindiati et al., 2007; P. J. Rousseeuw & Van Driessen, 1999). Rousseeuw and Van Driessen (1999) found that accurate results for small data sets can be obtained by FMCD in a little time. Additionally, FMCD gives more accurate results than other algorithms for large data sets (P. J. Rousseeuw & Van Driessen, 1999).

However, FMCD still not computationally efficient for large data sets with high dimensions

(Herwindiati et al., 2007). The computational complexity increases exponentially when the dimension, p increases (Djauhari, 2008). These estimators are constructed based on Covariance Determinant (CD) that have singularity problems (Herwindiati et al., 2007; Rohayu, 2013). Hence, to overcome the singularity problem, Herwindiati et al. (2007) proposed Minimum Vector Variance (MVV) which minimized Vector Variance (VV) instead of CD. Unlike CD, the computation of VV is simple and efficient, covariance does not need to be positive definite and applicable to high-dimension data sets (Herwindiati et al., 2007). Additionally, MVV is robust and has the same breakdown point as the MVE and MCD-based methods (Herwindiati et al., 2007). Herwindiati et al. (2007) compare the performance of MVE, FMCD and MVV in detecting outliers. All the methods can detect all outliers accurately and show a clear separation between outliers and inliers (Herwindiati et al., 2007). In terms of computational complexity, VV is found to have a significantly smaller computation speed than CD (Herwindiati et al., 2007).

Despite the advantages of MVV, it is still time-consuming and the running time increase as the dimension, p increase (Salleh & Djauhari, 2011). To overcome this problem, Rohayu (2013) developed two robust estimators which are Covariance Matrix Equality (CME) and Index Set Equality (ISE). It is found that CME and ISE are as effective as FMCD and MVV and computationally more efficient since CME and ISE are innovation methods from FMCD (Lim & Midi, 2016). For CME, the equality of covariance matrix $S_{H_{old}}$ and $S_{H_{new}}$ is tested while for ISE, the equality of data subset I_{old} and I_{new} is tested (Lim & Midi, 2016). Although ISE had better performance than other robust estimators, it only involves logical comparison and does not have any computations. Based on the idea of CME and ISE, a new robust estimator called Test on Covariance (TOC) is proposed by Abd Mutalib et al. (2019). TOC test the equality of two covariance structures. Although Abd Mutalib et al. (2019) had proven TOC is applicable to detect outliers in multivariate data and is a promising approach, more studies need to be done to observe the applicability of TOC in various conditions.

3. Methodology

In this section, five robust estimators are presented. The outlier scenarios, simulation study and performance measures will be explained in this section.

3.1 Algorithm for Existing Robust Estimators

Algorithms for FMCD, MVV, CME, ISE and TOC are discussed in this section. Algorithms for MVV, CME, ISE and TOC are innovation methods derived from the FMCD algorithm. The only difference between these methods is the stopping rule in Step 6. The algorithm for FMCD is given as follows (Lim & Midi, 2016).

Step 1: Select an arbitrarily subset H_{old} containing h different observations where h is the smallest integer $\geq (n+p+1)/2$, where p is the number of variable and n is the sample size.

Step 2: Compute the mean vector $\bar{X}_{H_{old}}$ and covariance matrix $S_{H_{old}}$ of all observations belonging to H_{old}.

Step 3: Compute $d^2_{H_{old}}(i) = \left(X_i - \bar{X}_{H_{old}}\right)' S^{-1}_{H_{old}} \left(X_i - \bar{X}_{H_{old}}\right)$ for $i = 1,2,\ldots,n$.

Step 4: Sort $d^2_{H_{old}}(i)$ for $i = 1,2,\ldots,n$ in increasing order $d^2_{H_{old}}(\pi(1)) \leq d^2_{H_{old}}(\pi(2)) \leq \ldots \leq d^2_{H_{old}}(\pi(n))$ where π is a permutation on $\{1,2,\ldots,n\}$. $d^2_{H_{old}}(\pi(1)) \leq d^2_{H_{old}}(\pi(2)) \leq \ldots \leq d^2_{H_{old}}(\pi(n))$ is an ordered distance.

Step 5:Define $H_{new} = \{X_{\pi(1)}, X_{\pi(2)}, \ldots, X_{\pi(h)}\}$ and then calculate $\bar{X}_{H_{new}}$, $S_{H_{new}}$ and $d^2_{H_{new}}(i)$ for $i = 1,2,\ldots,n$.

Step 6FMCD: If $\det(S_{H_{new}}) = 0$, repeat Step 1 – Step 5. Otherwise, if $\det(S_{H_{new}}) < \det(S_{H_{old}})$, let $H_{old} := H_{new}$, $\bar{X}_{H_{old}} := \bar{X}_{H_{new}}$ and $S_{H_{old}} := S_{H_{new}}$. Then go to Step 3. Otherwise, the process is stopped and $\det(S_{H_{new}}) = \det(S_{H_{old}})$ is obtained.

The algorithms for MVV, ISE and CME are the same as the FMCD algorithm except for Step 6 (Salleh & Djauhari, 2011; Lim & Midi, 2016).

Step 6MVV: If $Tr(S^2_{H_{new}}) = 0$, repeat Step 1 – Step 5. Otherwise, if $Tr(S^2_{H_{new}}) \neq Tr(S^2_{H_{old}})$, let $H_{old} := H_{new}$, $\bar{X}_{H_{old}} := \bar{X}_{H_{new}}$ and $S_{H_{old}} := S_{H_{new}}$ .Then go to Step 3. Otherwise, the process is stopped and $Tr(S^2_{H_{new}}) = Tr(S^2_{H_{old}})$ is obtained.

Step 6CME: If $\sqrt{Tr\left(S_{H_{new}} - S_{H_{old}}\right)^2} \neq 0$, calculates $\bar{X}_{H_{new}}$ and let $H_{old} := H_{new}$, $\bar{X}_{H_{old}} := \bar{X}_{H_{new}}$ and $S_{H_{old}} := S_{H_{new}}$. Then go to Step 3. Otherwise, the process is stopped.

Step 6ISE: If $I_{new} \neq I_{old}$, let $H_{old} := H_{new}$, calculate $\bar{X}_{H_{new}}$ and let $H_{old} := H_{new}$, $\bar{X}_{H_{old}} := \bar{X}_{H_{new}}$ and $S_{H_{old}} := S_{H_{new}}$. Then go to Step 3. Otherwise, the process is stopped.

It has been proven that ISE is effective as FMCD and MVV in finding robust estimators of mean and covariance matrix with lower computational time (Salleh & Djauhari, 2011; Lim & Midi, 2016). ISE is the only logical comparison of two index sets which are the old subset and the new subset. The comparison is made to determine the equality of covariance structure for both subsets.

By adopting the idea of ISE, the Test on Covariance (TOC) involving the equality test of a variance-covariance structure is proposed. The equality of two covariance structure is tested by using Eq. 2 with the hypothesis $H_0 : \Sigma_{old} = \Sigma_{new}$ versus $H_1 : \Sigma_{old} \neq \Sigma_{new}$,

$$u = \upsilon\left[\sum_{i=1}^{p}(\lambda_i - \ln\lambda_i) - p\right] \tag{2}$$

where $\upsilon = n-1$, $p = 1,2,\ldots,k$ and $\lambda_1,\lambda_2,\ldots,\lambda_p$ are the eigenvalues of $\Sigma_{new}\Sigma_{old}^{-1}$. H_0 is rejected if $u > \chi^2\left[\alpha, \frac{1}{2}p(p+1)\right]$ (Rencher, 2002).

The algorithm for TOC is the same as FMCD except a new procedure is proposed for Step 6.

Step 6$_{TOC}$: If H_0 is rejected, calculate $\bar{X}_{H_{new}}$ and let $H_{old} := H_{new}$, $\bar{X}_{H_{old}} := \bar{X}_{H_{new}}$ and $S_{H_{old}} := S_{H_{new}}$. Then go to Step 3. Otherwise, the process is stopped.

3.2 Outlier Scenarios

Random data for simulation studies are generated from the following mixture p-variate normal distributions (Filzmoser, 2005; Filzmoser et al., 2008; Herwindiati, 2007; Andrea et al., 2011; Cabana et al., 2017) and is given as,

$$(1-\varepsilon)N_p(\vec{\mu}_0,\Sigma_0) + \varepsilon N_p(\lambda\vec{\mu}_1,\delta\Sigma_1) \tag{3}$$

where $\Sigma_0 = \Sigma_1 = I_p$, $\vec{\mu}_0 = (0\ 0\ldots 0)'$ and $\vec{\mu}_1 = (1\ 1\ldots 1)'$ is of dimension p. Inliers are generated from $N_p(\vec{\mu}_0,\Sigma_0)$, whereas outliers are generated from $N_p(\lambda\vec{\mu}_1,\delta\Sigma_1)$. The percentage of outliers ε will be used to determine the number of outliers in the data. For example, if the data has a sample size $n = 100$ and a percentage of outliers $\varepsilon = 0.05$ is used, there will be five outliers in the data.

From the mixture distribution in Eq. 3, the coefficients λ and δ determine the outliers' scenarios. Two outliers' scenarios which are the mean-shift model and variance-inflation model (Pan et al., 2000) are considered in this study. The following are the explanation of each outliers' scenarios.

i. **Outlier scenario 1: Mean-shift model**

Mean-shift model have also been named as shifts of location, shift outliers, or location outliers (Pan et al., 2000; Filzmoser, 2005). This scenario will be determined by the value of λ whereas the value of δ is 1 and is given as follows,

$$(1-\varepsilon)N_p(\vec{\mu}_0,\Sigma_0) + \varepsilon N_p(\lambda\vec{\mu}_1,\Sigma_1). \tag{4}$$

The values λ that are used in this study are $\lambda = 1, 2, 4$ and 10. This range of values had been used in the previous study such as by Filzmoser et al. (2008), Andrea et al. (2011) and Cabana et al. (2017). The values λ show that the distance between outliers and inliers determining by shifting the mean.

ii. **Outlier scenario 2: Variance-inflation model**

The second outliers' scenario has also been named as shift of scale or scatter outliers (Pan et al., 2000; Filzmoser et al., 2008). This scenario is determined by the value of δ whereas the value of λ is 0 and is given as follows,

$$(1-\varepsilon)N_p(\vec{\mu}_0, \Sigma_0) + \varepsilon N_p(0, \delta\Sigma_1). \tag{5}$$

The values δ that are chosen in this study are $\delta = 0.1, 0.5, 2, 10$ and 25 as these values also had been used in Werner (2003), Filzmoser et al. (2008) and Cabana et al. (2017) studies. The values δ show that the distance between outliers and inliers determining by shifting the variance.

3.3 Simulation study and Performance Measures

In this study, we generate three sample sizes $n = 30, 100$ and 1000 representing small, moderate, and large sample sizes. For each sample size, there are three different number of variables, $p = 2, 3$ and 5. The percentage of outliers are $\varepsilon = 0.05, 0.15$ and 0.25. The selected sample sizes, number of variables and percentage of outliers are adopted from Rocke and Woodruff (1996), Kosinski (1999), Filzmoser et al. (2008), Herwindiati et al. (2007), Fauconnier and Haesbroeck (2009) and Andrea et al. (2011). Each combination of n, p and ε is set for two outlier scenarios. The simulation study is done by using the R version 4.0.0 statistical package. In this study the number of simulations, s is set to be 10000. Table 1 summarize the design specification of the simulation study.

Table 1. The combination values of n, p and ε for two outlier scenarios.

Sample Size, n	**Number of Variables,** p	**Percentage of outliers,** ε
30	2, 3, 5	0.05, 0.15, 0.25
100	2, 3, 5	0.05, 0.15, 0.25
1000	2, 3, 5	0.05, 0.15, 0.25

The performance of the proposed methods is measured by three different measurements which are success probability (*pout*), masking effect (*pmask*), and swamping effect (*pswamp*).

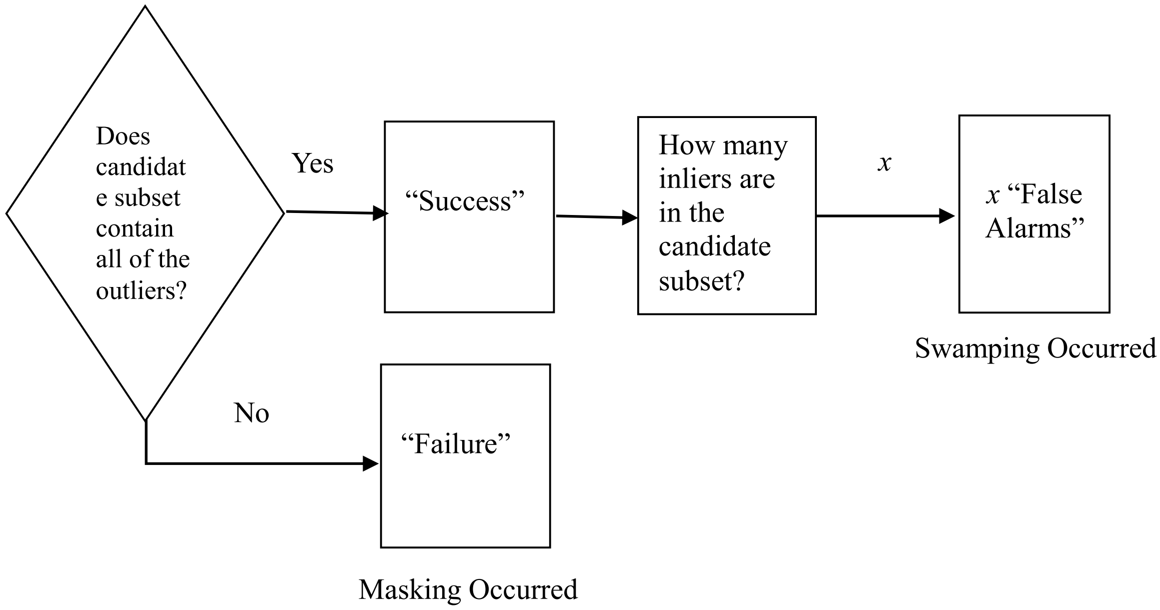

Figure 1. Flowchart for the assessment of the proposed method (Sebert et al.,1998)

Sebert et al. (1998) illustrated how the performance of the methods to detect outliers was measured. "Success" indicates the proposed methods can detect all the outliers and "Failure" happens when the proposed methods fail to detect all the outliers. If the proposed method can detect all the outliers but contains inliers, a "false alarm" would occur. Eq. 6- 8 are the formulas for each performance measure.

i. *pout* is the probability that all the outliers are successfully detected given by

$$pout = \frac{"success"}{s} \tag{6}$$

where "*success*" is the number of data set that the robust estimators successfully identified all the outliers and *s*is the total number of simulations.

ii. *pmask* is the probability that the outliers are falsely detected as inliers and given by

$$pmask = \frac{"failure"}{(out)(s)} \tag{7}$$

where "*failure*" is the number of outliers in all data set that detected as inliersand *out* is the number of outliers.

iii. *pswamp* is the probability of inliers detected as outliers and given by

$$pswamp = \frac{"false"}{(n-out)(s)} \tag{8}$$

where "*false*" is the number of inliers in all data set that detected as outliers, *x* and *n* is the sample size.

The values of *pout*, *pmask* and *pswamp* will be between 0 and 1. The best method will show the highest value of *pout* when the value approaching 1 and the lowest value of *pmask* and *pswamp* when the value approaching 0 (Santos-Pereira & Pires, 2002).

4. Results and Discussion

In this section, results for two outlier scenarios will be presented and discussed. The performance of each robust estimator according to the *pout*, *pmask* and *pswamp* values are discussed.

4.1 Outlier Scenario 1: Mean-shift model

Results of the simulation study for Outlier Scenario 1 are presented in Tables 2 - 4. Table 2 shows values of *pout* for 5%, 15% and 25% of outliers. From Table 2, the values of *pout* for all robust estimators increase when the distance of outliers, λ from the inliers, increase for all sample sizes and number of variables, p. It shows that all robust estimators have better performance to detect outliers when the distance between outliers and inliers gets larger. For all percentage of outliers, sample sizes, the number of variables and $\lambda = 10$, all robust estimators obtained *pout* values of 1.0000. This result shows that all robust estimators can detect all outliers successfully. This is happening because the distance between outliers and inliers is large. For $\lambda = 4$, results also show certain cases with *pout* values 1.0000 and the rest of pout values more than 0.9000 except for 25% of outliers with $n = 30$ and $p = 5$.

There are also *pout* values 0.0000 such as for $n = 1000$, $\lambda = 1$, all percentage of outliers and for all number of variables. It shows that all robust estimators failed to detect all outliers. This can happen because of the distance of outliers and inliers as well as when the number of outliers is small. As can be seen in $n = 1000$, $\lambda = 1$ and $\varepsilon = 0.05$, this data only contain 50 outliers and the distance between outliers and inliers is small and this condition makes all robust estimators failed to detect outliers.

Table 2. *pout* values for outlier scenarios 1.

p	n	λ	$\varepsilon = 5\%$					$\varepsilon = 15\%$					$\varepsilon = 25\%$				
			FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC
2	30	1	**0.1744**	**0.1744**	**0.1744**	**0.1744**	**0.1744**	0.0043	0.0057	**0.0069**	0.0057	0.0043	0.0012	**0.0086**	**0.0086**	0.0051	0.0007
		2	**0.7977**	0.6284	**0.7977**	0.6284	0.7104	**0.3901**	**0.3901**	**0.3901**	**0.3901**	**0.3901**	**0.3481**	0.2308	0.2593	**0.3481**	0.2593
		4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9953	**0.9980**	**0.9980**	0.9953	0.9953	**0.9999**	**0.9999**	**0.9999**	**0.9999**	**0.9999**
		10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
	100	1	0.0106	0.0093	0.0106	0.0106	**0.0164**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.3316	0.3310	0.3310	0.3310	**0.3436**	**0.0513**	**0.0513**	0.0398	0.0504	**0.0513**	0.0102	**0.0565**	**0.0565**	0.0060	0.0092
		4	**0.9996**	**0.9996**	**0.9996**	0.9994	0.9993	**0.9977**	**0.9977**	**0.9977**	**0.9977**	**0.9977**	**0.9983**	**0.9983**	**0.9983**	**0.9983**	0.9855
		10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
	1000	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		4	**0.9922**	**0.9922**	**0.9922**	**0.9922**	0.9884	**0.9771**	**0.9771**	**0.9771**	**0.9771**	**0.9771**	**0.9976**	**0.9976**	**0.9976**	**0.9976**	0.9964
		10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	30	1	**0.3385**	0.2045	0.2588	0.1921	0.1921	0.0044	**0.0057**	0.0036	**0.0057**	0.0044	0.0145	**0.0147**	0.0061	0.0059	0.0145
		2	**0.9835**	0.9298	0.9298	0.9298	0.9587	**0.4014**	0.1247	**0.4014**	0.1247	**0.4014**	0.6950	0.2704	0.6950	**0.7369**	0.1766
		4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9999
		10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
	100	1	0.0055	**0.0069**	**0.0069**	0.0063	0.0055	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	**0.7383**	**0.7383**	**0.7383**	**0.7383**	**0.7383**	**0.0305**	0.0146	**0.0305**	**0.0305**	**0.0305**	0.1005	0.1151	**0.1292**	0.1151	0.1058
		4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9987	**0.9988**	0.9987	0.9987	0.9987	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
	1000	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	**0.0103**	**0.0103**	**0.0103**	0.0101	0.0045	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**0.9995**	**0.9995**	**0.9995**	**0.9995**	**0.9995**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
5	30	1	0.4699	0.4054	0.4081	**0.5278**	0.4571	**0.1672**	0.0858	0.0881	0.0798	0.0932	0.0286	0.0286	0.0138	**0.0378**	0.0099
		2	0.7886	**0.7919**	0.7611	0.7683	0.7872	0.5131	0.5075	**0.5259**	0.5022	0.1004	0.0397	0.0303	0.0364	0.0447	**0.0523**
		4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0447	0.0826	**1.0000**	0.2121	0.1629
		10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.3824	**1.0000**	**1.0000**	**1.0000**	0.2146
	100	1	0.0355	**0.0485**	0.0299	0.0302	0.0303	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.9411	**0.9487**	0.9411	0.9412	0.9467	**0.2876**	**0.2876**	0.2741	**0.2876**	0.1523	**0.5332**	0.3803	0.2393	0.1400	**0.5332**
		4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
	1000	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.2456	0.2411	0.2445	**0.2487**	0.2094	0.0023	0.0023	**0.0024**	0.0023	0.0015	**0.0115**	0.0059	**0.0115**	0.0059	0.0000
		4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**

From Table 2, it is found that TOC gives better or equal results to detect outliers as other robust estimators. For 5% outliers, FMCD, ISE and TOC give better results than other robust estimators for 2 out of 36 cases. While MVV gives better results than other robust estimators for 3 out of 36 cases. FMCD and MVV also show 22 out of 36 cases give an equal performance with other robust estimators. Meanwhile, CME, ISE and TOC give an equal performance with other robust estimators for 23, 19 and 18 out of 36 cases, respectively.

For 15% outliers, it is found that CME gives better results than other robust estimators for 3 out of 36 cases. FMCD and MVV show that 1 out of 36 cases give better results than other robust estimators. Meanwhile, for equal performance to detect outliers, FMCD and MVV show 21 out of 36 cases has the same results as other robust estimators. CME, ISE and TOC give an equal performance with other robust estimators for 20 out of 36 cases. For 25% outliers, CME and ISE show 2 out of 36 cases give better results than other robust estimators. While MVV and TOC show 1 out of 36 cases give better results than other robust estimators. Results for equal performance to detect outliers show FMCD and MVV has 19 out of 36 cases has same results with other robust estimators. Meanwhile CME, ISE and TOC show 20, 18 and 14 out of 36 cases give an equal performance with other robust estimators.

Tables 3 show values of *pmask* for 5%, 15% and 25% percentage of outliers. From Tables 3, the values of *pmask* for all robust estimators decrease as the number of variables increases except for certain cases. Results also show that the *pmask* values of all robust estimators decrease as the values λ increase. It shows that as the distance of outliers increases, the performance of robust estimators is better.

For $\lambda = 10$, *pmask* values is 0.0000 show that all robust estimators do not misclassify outliers as inliers. This happens because the distance between outliers and inliers is large. However, for the case of 25% outliers, the values of *pmask* for $n = 30,\ p = 5$ is not 0.0000. Results $\lambda = 4$ also show *pmask* value of 0.0000 and approaching 0.0000 for most of the other cases. It is found that for 5% outliers, FMCD and ISE give better results than other robust estimators for 2 out of 36 cases. While MVV and TOC give better results than other robust estimators for 4 out of 36 cases. FMCD and MVV also show 23 out of 36 cases give an equal performance with other robust estimators. Meanwhile, CME, ISE and TOC give an equal performance with other robust estimators for 24, 21and 20 out of 36 cases, respectively.

For 15% outliers, it is found that CME gives better results than other robust estimators for 4 out of 36 cases. FMCD and TOC show that 2 out of 36 cases give better results than other robust estimators. While MVV gives better results than other robust estimators for 1 out of 36 cases. For equal performance to not misclassify outliers as inliers, FMCD and MVV show 25 out of 36 cases has the same results with other robust estimators. CME, ISE and TOC give an equal performance with other robust estimators for 24, 23 and 18 out of 36 cases, respectively.

For 25% outliers, CME show 5 out of 36 cases give better results than other robust estimators. ISE shows that 2 out of 36 cases give better results than other robust estimators. While FMCD, MVV and TOC show 1 out of 36 cases give better results than other robust estimators. Results for an equal performance show that MVV and CME have 22 out of 36 cases that givethe same results as other robust estimators. Meanwhile FMCD, ISE and TOC show 23, 21 and 16 out of 36 cases give an equal performance with other robust estimators.

Table 3. *pmask* values for outlier scenarios 1.

p	n	λ	$\varepsilon = 5\%$					$\varepsilon = 15\%$					$\varepsilon = 25\%$				
			FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC
2	30	1	**0.5797**	**0.5797**	**0.5797**	**0.5797**	**0.5797**	0.6703	0.6408	**0.6203**	0.6408	0.6408	0.5789	**0.4442**	**0.4442**	0.4790	0.6180
		2	**0.1068**	0.2092	**0.1068**	0.2092	0.1574	**0.1708**	**0.1708**	**0.1708**	**0.1708**	**0.1708**	**0.1239**	0.1682	0.1557	**0.1239**	0.1557
		4	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.0009	**0.0004**	**0.0004**	0.0009	**0.0004**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
		10	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
	100	1	0.5887	0.6043	0.5887	0.5887	**0.5602**	**0.5698**	**0.5698**	**0.5698**	**0.5698**	0.5893	0.6171	**0.4799**	0.4807	0.5801	0.6138
		2	0.1980	0.1985	0.1985	0.1985	**0.1921**	**0.1787**	**0.1787**	0.1892	0.1790	0.1943	0.1719	**0.1094**	**0.1094**	0.1857	0.1727
		4	**0.0001**	**0.0001**	**0.0001**	**0.0001**	**0.0001**	0.0002	0.0002	0.0002	0.0002	**0.0001**	**0.0001**	**0.0001**	**0.0001**	**0.0001**	0.0006
		10	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
	1000	1	0.6483	0.6483	0.6483	0.6483	**0.6433**	**0.7140**	**0.7140**	**0.7140**	**0.7140**	0.7184	**0.5619**	**0.5619**	**0.5619**	**0.5619**	0.5700
		2	**0.2031**	**0.2031**	**0.2031**	**0.2031**	0.2130	**0.2672**	**0.2672**	**0.2672**	**0.2672**	0.3271	**0.1066**	**0.1066**	**0.1066**	**0.1066**	0.2572
		4	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.0001
		10	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
3	30	1	**0.4192**	0.5468	0.4904	0.5594	0.5594	0.6808	**0.6567**	0.6836	**0.6567**	**0.6567**	**0.4137**	0.4150	0.4817	0.4816	**0.4137**
		2	**0.0082**	0.0355	0.0355	0.0355	0.0208	**0.1670**	0.3411	**0.1670**	0.3411	0.3655	0.0443	0.1524	0.0443	**0.0374**	0.1959
		4	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
		10	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
	100	1	0.6438	**0.6212**	**0.6212**	0.6378	0.6410	**0.5398**	**0.5398**	**0.5398**	0.5440	0.5546	0.4188	0.4039	0.4146	**0.3816**	0.4146
		2	**0.0593**	**0.0593**	**0.0593**	**0.0593**	**0.0593**	**0.2058**	0.2408	**0.2058**	**0.2058**	0.2108	0.0870	0.0827	**0.0782**	0.0827	0.0857
		4	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.0001	0.0001	0.0001	0.0001	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
		10	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
	1000	1	0.6186	0.6173	0.6189	0.6173	**0.6171**	0.7118	**0.7116**	0.7118	0.7118	0.7155	0.5266	0.5292	**0.5245**	0.5292	0.5404
		2	**0.0859**	**0.0859**	**0.0859**	0.0861	0.1011	**0.1739**	**0.1739**	**0.1739**	**0.1739**	0.2124	**0.0833**	**0.0833**	0.0836	**0.0833**	0.2180
		4	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
		10	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
5	30	1	0.3122	0.3640	0.3614	**0.2735**	0.3239	**0.3027**	0.3905	0.3868	0.3959	0.3750	**0.3609**	0.3620	0.4099	0.3412	0.4379
		2	0.1122	**0.1106**	0.1281	0.1239	0.1133	0.1246	0.1260	**0.1201**	0.1285	0.3738	0.3365	0.3574	0.3427	0.3266	**0.3112**
		4	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.3173	0.2702	**0.0000**	0.1761	0.1999
		10	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.1130	**0.0000**	**0.0000**	**0.0000**	0.1745
	100	1	0.4846	**0.4526**	0.4968	0.4984	0.4986	**0.6143**	0.6216	0.6198	0.6198	0.6240	0.5005	0.5067	**0.4864**	0.4894	0.5031
		2	0.0120	**0.0104**	0.0120	0.0119	0.0108	**0.0803**	**0.0803**	0.0831	**0.0803**	0.1170	**0.0249**	0.0380	0.0557	0.0756	**0.0249**
		4	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
		10	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
	1000	1	0.5426	**0.5405**	0.5426	0.5432	0.5438	0.7376	0.7320	**0.7313**	0.7376	0.7337	0.5927	0.5914	**0.5911**	0.5931	0.5929
		2	0.0277	0.0280	0.0277	**0.0274**	0.0308	0.0393	0.0393	**0.0392**	0.0393	0.0416	**0.0169**	0.0192	**0.0169**	0.0192	0.3650
		4	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
		10	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Table4 shows values of *pswamp* for 5%, 15% and 25% percentage of outliers.Overall, the *pswamp* values increase as the number of variables increases for all robust estimators except for certain cases. However, for 15% outliers, *pswamp* values increase as the number of variables increases for all robust estimators except for $n = 1000$.Results also show the probability that all robust estimators misclassify inliers as outliers decrease as the values λ increase. It shows that as the distance between outliers and inliers increases, the performance of robust estimators is better. TOC gives better results for most of the cases for 5%, 15% and 25% of outliers.

It is found that for 5% outliers, TOC gives better results than other robust estimators for 21 out of 36 cases. MVV and ISE give better results than other robust estimators for 3 out of 36 cases. While FMCD gives better performance with other robust estimators for 1 out of 36 cases. For equal performance to not misclassify inliers as outliers, FMCD and ISE show 7 out of 36 cases have the same results with other robust estimators. CME and TOC give an equal performance with other robust estimators for 6 out of 36 cases. Meanwhile, MVV gives an equal performance with other robust estimators for 8 out of 36 cases.

For 15% outliers, it is found that TOC gives better results than other robust estimators for 16 out of 36 cases. FMCD and CME show that 2 out of 36 cases give better results than other robust estimators. While MVV and ISE give better results than other robust estimators for 3 and 1 out of 36 cases. For equal performance to not misclassify inliers as outliers, FMCD, CME and ISE show 10 out of 36 cases have the same results with other robust estimators. While MVV and TOC each give an equal performance with other robust estimators for 9 and 6 out of 36 cases, respectively.

For 25% outliers, TOC shows12 out of 36 cases give better results than other robust estimators. FMCD and ISE show 3 out 36 cases give better results than other robust estimators. While MVV and CME show 2 out of 36 cases give better results than other robust estimators. Results for equal performance show CME and ISE has 10 out of 36 cases give same results with other robust estimators. Meanwhile, FMCD, MVV and TOC show 9, 11 and 6 out of 36 cases give an equal performance with other robust estimators.

Table 4. *pswamp* values for outlier scenarios 1.

p	n	λ	$\varepsilon = 5\%$					$\varepsilon = 15\%$					$\varepsilon = 25\%$				
			FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC
2	30	1	**0.1700**	**0.1700**	**0.1700**	**0.1700**	**0.1700**	**0.1213**	0.1361	0.1258	0.1361	0.1361	0.2978	**0.2960**	**0.2960**	0.3070	0.3181
		2	0.1331	0.1423	0.1331	0.1423	**0.1201**	**0.1111**	**0.1111**	**0.1111**	**0.1111**	**0.1111**	0.2469	**0.2327**	0.2453	0.2469	0.2453
		4	0.1168	**0.1066**	0.1117	0.1117	0.1117	0.0978	**0.0769**	**0.0769**	0.0978	**0.0769**	**0.1573**	**0.1573**	**0.1573**	**0.1573**	**0.1573**
		10	**0.0749**	**0.0749**	**0.0749**	**0.0749**	**0.0749**	**0.0574**	0.0580	0.0580	0.0636	0.0580	**0.1467**	**0.1467**	**0.1467**	**0.1467**	**0.1467**
	100	1	0.1941	0.1980	0.1941	0.1941	**0.1910**	0.1737	0.1737	0.1737	0.1737	**0.1710**	0.2192	0.2301	0.2339	0.2172	**0.2160**
		2	0.1560	0.1549	0.1549	0.1549	**0.1509**	0.1365	0.1365	0.1372	**0.1294**	0.1299	**0.1763**	0.1956	0.1956	0.1826	0.1790
		4	0.1085	0.1085	0.1085	0.1116	**0.0968**	0.0841	0.0841	0.0941	0.0841	**0.0799**	0.1260	0.1260	0.1260	0.1260	**0.1224**
		10	0.0927	0.0927	0.0927	0.0927	**0.0740**	**0.0593**	**0.0593**	**0.0593**	**0.0593**	**0.0593**	0.1213	0.1213	0.1213	0.1213	**0.0924**
	1000	1	0.1337	0.1337	0.1337	0.1337	**0.1315**	0.1305	0.1305	0.1305	0.1305	**0.1300**	**0.2545**	**0.2545**	**0.2545**	**0.2545**	0.2562
		2	**0.1286**	**0.1286**	**0.1286**	**0.1286**	0.1287	**0.0903**	**0.0903**	**0.0903**	**0.0903**	0.0953	**0.2016**	**0.2016**	**0.2016**	**0.2016**	0.2168
		4	**0.1153**	**0.1153**	**0.1153**	**0.1153**	0.1169	0.0719	0.0719	0.0719	0.0719	**0.0710**	0.2012	0.2012	0.2012	0.2012	**0.1977**
		10	0.1111	0.1111	0.1111	0.1111	**0.1103**	**0.0662**	**0.0662**	**0.0662**	**0.0662**	0.0666	**0.1818**	**0.1818**	**0.1818**	**0.1818**	0.1838
3	30	1	0.2834	**0.2472**	0.2569	0.2548	0.2548	0.1985	0.1867	**0.1816**	0.1867	0.1867	**0.4109**	0.4318	0.4120	0.4284	**0.4109**
		2	0.2260	0.2446	0.2446	0.2446	**0.1924**	**0.1414**	0.1516	**0.1414**	0.1516	0.1551	0.3121	0.3250	0.3121	0.2790	**0.2745**
		4	0.1699	0.1699	0.1607	0.1699	**0.1526**	**0.1131**	0.1262	0.1262	**0.1131**	**0.1131**	0.1218	**0.1043**	**0.1043**	**0.1043**	0.1213
		10	0.1041	0.1093	0.0976	0.0976	**0.0769**	**0.0684**	**0.0684**	**0.0684**	**0.0684**	**0.0684**	**0.0814**	**0.0814**	**0.0814**	**0.0814**	**0.0814**
	100	1	**0.1702**	0.1802	0.1802	0.1807	0.1767	0.1486	0.1486	0.1486	0.1538	**0.1439**	0.3317	0.3231	**0.3146**	0.3199	**0.3146**
		2	**0.1559**	**0.1559**	**0.1559**	**0.1559**	**0.1559**	0.1272	0.1320	0.1272	0.1272	**0.1185**	0.2800	0.2801	**0.2676**	0.2801	0.2863
		4	**0.1246**	**0.1246**	0.1269	**0.1246**	**0.1246**	0.0822	0.0822	0.0810	0.0822	**0.0793**	**0.2084**	0.2129	0.2129	0.2107	0.2148
		10	0.1097	0.1097	0.1150	0.1097	**0.0960**	0.0714	0.0714	0.0714	0.0714	**0.0690**	0.1854	0.1854	0.1854	**0.1578**	0.1788
	1000	1	0.1224	0.1224	0.1226	0.1224	**0.1223**	0.1181	0.1183	0.1181	0.1181	**0.1180**	0.2538	0.2543	0.2528	0.2543	**0.2499**
		2	0.1108	0.1108	0.1108	0.1110	**0.1097**	**0.0719**	**0.0719**	**0.0719**	**0.0719**	0.0739	**0.1775**	**0.1775**	**0.1775**	**0.1775**	0.1962
		4	0.1062	0.1062	0.1065	0.1062	**0.1060**	0.0702	0.0702	0.0702	0.0702	**0.0691**	0.1684	0.1684	0.1696	0.1696	**0.1626**
		10	0.1036	0.1037	0.1037	**0.1029**	0.1039	0.0683	0.0683	0.0683	0.0683	**0.0680**	0.1568	0.1568	0.1568	0.1568	**0.1530**
5	30	1	0.2814	0.3067	0.3215	**0.2592**	0.2735	0.5653	**0.4629**	0.4855	0.4721	0.5571	0.6865	0.6687	0.6018	**0.5593**	0.5892
		2	0.1828	0.1901	0.2022	0.2154	**0.1798**	0.3618	**0.3428**	0.3768	0.3534	0.3467	0.4665	**0.4577**	0.4969	0.5157	0.4973
		4	0.1576	**0.1477**	0.1560	0.1576	**0.1477**	0.2902	**0.2415**	0.2902	**0.2415**	0.2902	0.3888	0.4295	**0.3713**	0.4028	0.4584
		10	0.1291	0.1202	0.1443	0.1508	**0.0757**	0.2201	0.2201	0.2201	0.2201	**0.1737**	**0.2790**	0.4082	0.3249	0.3954	0.4084
	100	1	0.1916	0.1933	0.1822	**0.1777**	0.1778	0.1747	0.1763	0.1740	0.1740	**0.1614**	0.4335	0.4349	0.4443	0.4471	**0.4286**
		2	0.1598	0.1611	0.1598	0.1629	**0.1554**	0.1395	0.1395	**0.1338**	0.1395	0.1400	**0.2792**	0.2880	0.3065	0.3200	**0.2792**
		4	0.0997	0.0961	0.0968	0.0933	**0.0848**	**0.1137**	0.1187	**0.1137**	**0.1137**	0.1187	0.2416	0.2651	0.2499	**0.2375**	0.2416
		10	0.0710	**0.0700**	0.0751	0.0751	0.0757	0.0976	**0.0938**	0.0960	0.1026	0.0961	0.2369	**0.2099**	0.2357	**0.2099**	0.2140
	1000	1	0.1111	0.1112	0.1111	0.1112	**0.1109**	0.0860	0.0855	0.0855	0.0860	**0.0852**	0.2522	0.2523	0.2514	0.2523	**0.2425**
		2	0.1028	0.1028	0.1027	0.1028	**0.1018**	**0.0694**	**0.0694**	**0.0694**	**0.0694**	**0.0694**	0.1640	**0.1639**	0.1640	**0.1639**	0.2036
		4	**0.1005**	**0.1005**	**0.1005**	**0.1005**	**0.1005**	0.0689	0.0689	0.0689	0.0689	**0.0687**	0.1611	0.1611	0.1610	0.1611	**0.1598**
		10	0.0984	0.0984	0.0982	0.0982	**0.0971**	0.0644	0.0644	0.0644	0.0644	**0.0640**	0.1367	0.1365	0.1367	0.1367	**0.1350**

4.2 Outlier Scenario 2: Variance-inflation model

Results of the simulation study for Outlier Scenario 2 are presented in this section from Table 5 to Tab le 7. Table 5 shows values of *pout* for 5%, 15% and 25% percentage of outliers. From Table 5, the values of *pout* for all robust estimators increase when the distance of outliers, δ increases for all sample sizes and all number of variables, p. It shows that all robust estimators have better performance to detect outliers when the distance between outliers and inliers gets larger. Results also show that *pout* values increase as the number of variables increase.

From Table 5, all robust estimators obtained *pout* values of 0.0000for all sample sizes, all number of variables and $\delta = 0.1$. The values of all robust estimators were also found to be 0.0000 for $\delta = 0.5$, $n = 100$ and $n = 1000$. The same result was obtained for $\delta = 2$ and $n = 1000$. This can happen as the distance between outliers and inliers is smaller compare to $\delta = 10$ and $\delta = 25$. From Table 5, *pout* values are 0.0000 for all sample sizes, all number of variables, $\delta = 0.1$ and $\delta = 0.5$. The same result was also obtained for $\delta = 2$, $n = 100$ and $n = 1000$.

Besides the distance of outliers, the percentage of outliers also affected the ability of robust estimators to detect outliers. If the data contain 5% outliers, only 2, 5 and 50 outliers contain in $n = 30$, $n = 100$ and $n = 1000$ which makes outliers identification difficult for all robust estimators. For 5% outliers, ISE gives better results than other robust estimators for 8 out of 45 cases. FMCD and MVV give better results than other robust estimators for 3 out of 45 cases. While TOC gives better results than other robust estimators for 1 out of 45 cases. FMCD and ISE also show 8 out of 45 cases give an equal performance with other robust estimators. Meanwhile, MVV, CME and TOC give an equal performance with other robust estimators for 9, 7 and 3 out of 45 cases, respectively.

For 15% outliers, it is found that CME and ISE give better results than other robust estimators for 3 out of 45 cases. FMCD and MVV show that 1 out of 45 cases give better results than other robust estimators. Meanwhile, for equal performance to detect outliers, FMCD and ISE show 7 out of 45 cases havethe same results with other robust estimators. MVV and CME show 6 out of 45 cases has the same results as other robust estimators. While TOC gives an equal performance with other robust estimators for 4 out of 45 cases.

For 25% outliers, FMCD and CME show 4and 5 out of 45 cases give better results than other robust estimators. While ISE and TOC show 1 out of 45 cases give better results than other robust estimators. Results for equal performance to detect outliers show MVV and CME has 4 out of 45 cases give same results with other robust estimators. Meanwhile, FMCD, ISE and TOC show 5, 6 and 3 out of 45 cases give an equal performance with other robust estimators.

Table 5. *pout* values for outlier scenarios 2.

p	n	δ	$\varepsilon = 5\%$					$\varepsilon = 15\%$					$\varepsilon = 25\%$				
			FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC
2	30	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.5	0.0004	**0.0009**	0.0004	0.0004	**0.0009**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.1000	0.1000	0.1000	**0.1052**	0.1000	**0.0070**	**0.0070**	0.0063	**0.0070**	0.0069	**0.0007**	0.0004	0.0004	0.0005	0.0004
		10	**0.6057**	**0.6057**	**0.6057**	**0.6057**	**0.6057**	0.3055	0.3007	0.3007	**0.3148**	0.3029	**0.2084**	**0.2084**	**0.2084**	**0.2084**	**0.2084**
		25	0.8144	0.8160	0.8144	**0.8175**	0.8160	0.6247	**0.6279**	0.6247	0.6247	0.6062	**0.4732**	0.4608	0.4623	0.4623	0.4541
	100	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.0096	**0.0099**	0.0096	**0.0099**	**0.0099**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		10	**0.3923**	**0.3923**	**0.3923**	**0.3923**	0.3764	**0.0297**	**0.0297**	**0.0297**	**0.0297**	0.0255	**0.0233**	0.0232	0.0196	0.0202	0.0202
		25	**0.6484**	0.6471	**0.6484**	**0.6484**	0.6471	0.1823	0.1823	**0.1830**	0.1823	**0.1830**	**0.1861**	**0.1861**	**0.1861**	**0.1861**	0.1845
	1000	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		10	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		25	**0.0124**	0.0122	**0.0124**	**0.0124**	0.0122	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	30	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.5	0.0006	0.0006	0.0006	0.0004	**0.0025**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	**0.1669**	**0.1669**	0.1514	**0.1669**	0.1662	**0.0103**	0.0100	0.0100	**0.0103**	**0.0103**	**0.0063**	**0.0063**	0.0062	0.0062	0.0022
		10	0.7919	0.7991	0.7991	**0.8066**	0.7919	0.4631	0.4631	**0.4770**	**0.4770**	**0.4770**	**0.4472**	0.4336	0.4336	**0.4472**	0.4270
		25	0.9267	**0.9280**	0.9267	0.9272	0.9233	**0.7731**	0.7693	0.7693	**0.7731**	0.7574	0.7249	0.7249	**0.7553**	0.7249	0.7175
	100	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.0242	0.0246	0.0246	**0.0271**	0.0244	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		10	**0.6001**	**0.6001**	**0.6001**	**0.6001**	0.5876	0.1496	0.1503	0.1496	**0.1513**	0.1485	0.0933	0.0998	**0.1020**	0.0960	0.0951
		25	0.8336	0.8329	0.8336	**0.8340**	0.8224	0.5308	**0.5332**	**0.5332**	0.5308	0.5182	0.4972	0.4946	**0.4974**	0.4946	0.4529
	1000	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		10	0.0022	**0.0023**	0.0022	0.0022	0.0021	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		25	**0.1806**	0.1798	0.1805	0.1805	0.1775	**0.0044**	**0.0044**	**0.0044**	**0.0044**	0.0043	**0.0029**	0.0028	**0.0029**	**0.0029**	0.0028
5	30	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.5	**0.0279**	0.0203	0.0185	0.0102	0.0052	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.4337	0.4445	0.4480	**0.4527**	0.4154	0.0368	0.0481	0.0381	**0.0620**	0.0450	0.0208	0.0168	0.0211	0.0123	**0.0222**
		10	0.9601	**0.9624**	**0.9624**	0.9613	0.9534	0.7854	0.7963	**0.8054**	0.7886	0.7963	0.6873	**0.7075**	**0.7075**	**0.7075**	**0.7075**
		25	0.9911	**0.9928**	0.9923	0.9913	0.9918	**0.9643**	**0.9643**	**0.9643**	**0.9643**	**0.9643**	0.9536	0.9541	0.9536	**0.9567**	0.9541

	100	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.0449	0.0449	0.0424	**0.0465**	0.0419	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0001**	0.0000	0.0000
		10	0.8461	0.8477	0.8458	**0.8480**	0.8454	0.4530	0.4533	**0.4679**	0.4568	0.4327	0.6431	0.6428	**0.6486**	0.6342	0.6462
		25	**0.9783**	**0.9783**	**0.9783**	**0.9783**	0.9777	0.9213	0.9202	**0.9216**	0.9202	0.9202	**0.9390**	0.9375	0.9362	0.9353	0.9353
	1000	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		10	**0.2121**	**0.2121**	0.2119	0.2119	0.2119	**0.0024**	0.0022	0.0022	0.0022	0.0022	0.0018	0.0019	0.0020	**0.0021**	**0.0021**
		25	**0.8066**	0.8062	0.8063	0.8060	0.8059	**0.4521**	**0.4521**	0.4518	0.4514	0.4508	0.4478	0.4485	0.4482	**0.4486**	0.4464

The probability that the outliers are falsely detected as inliers or known as *pmask* is shown in Table 6. From Table 6, the values of *pmask* for all robust estimators decrease as the number of variables increases except for certain cases $p = 5$. Results also show the *pmask* values decrease as the values δ increase. It shows that as the distance of outliers increases, the performance of robust estimators is better.

The values of *pmask* are 1.0000 for all sample sizes and $\delta = 0.1$ when data contain 5% and 15% outliers. However, when $n = 30$ and $\delta = 0.1$ for 5% outliers the values of *pmask* are 0.9000 and above. For 25% outliers, the value of *pmask* is 1.0000 for $n = 1000$ and $\delta = 0.1$. Results from Table 6 show that all robust estimators have the lowest probability to misclassify outliers as in inliers when $\delta = 10$ and $\delta = 25$.It is found that for 5% outliers, ISE gives better results than other robust estimators for 7 out of 45 cases. While FMCD, MVV, CME and TOC give better results than other robust estimators for 2, 5, 1 and 3 out of 45 cases. MVV and ISE also show 14 out of 45 cases give an equal performance with other robust estimators. Meanwhile, FMCD, CME and TOC give an equal performance with other robust estimators for 15, 16 and 6 out of 45 cases, respectively.

For 15% outliers, it is found that CME gives better results than other robust estimators for 7 out of 45 cases. FMCD, MVV and ISE show that 3 out of 45 cases give better results than other robust estimators. While TOC gives better results than other robust estimators for 1 out of 45 cases. For equal performance to not misclassify outliers as inliers, FMCD and MVV show 13 out of 45 cases has the same results with other robust estimators. CME, ISE and TOC give an equal performance with other robust estimators for 16, 15 and 8 out of 45 cases, respectively.

For 25% outliers, CME, FMCD and ISE show9, 6 and 5 out of 45 cases give better results than other robust estimators. While MVV and TOC show 1 out of 45 cases give better results than other robust estimators. Results for an equal performance show that FMCD and MVV have18 out of 45 cases that give the same results as other robust estimators. CME and ISE show 16 out of 45 cases have the same results as other robust estimators. Meanwhile, TOC show 4 out of 45 cases give an equal performance with other robust estimators.

Table 6. *pmask* values for outlier scenarios 2.

p	n	δ	$\varepsilon = 5\%$					$\varepsilon = 15\%$					$\varepsilon = 25\%$				
			FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC
2	30	0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9913**	0.9980	**0.9913**	0.9973	0.9973
		0.5	0.9790	**0.9745**	0.9790	0.9790	**0.9745**	**0.9574**	**0.9574**	**0.9574**	**0.9574**	0.9652	0.9076	0.9076	0.9124	**0.8954**	0.9076
		2	0.6850	0.6850	0.6850	**0.6759**	0.6850	0.6322	0.6322	**0.6302**	0.6322	0.6303	0.6060	**0.5943**	0.5948	0.6016	**0.5943**
		10	**0.2237**	**0.2237**	**0.2237**	**0.2237**	**0.2237**	0.2116	0.2142	0.2142	**0.2060**	0.2132	**0.1798**	**0.1798**	**0.1798**	**0.1798**	**0.1798**
		25	0.0974	0.0969	0.0974	**0.0956**	0.0969	0.0902	**0.0893**	0.0902	0.0902	0.0956	**0.0892**	0.0911	0.0915	0.0915	0.0934
	100	0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9914**	0.9969	**0.9914**	**0.9914**	0.9974
		0.5	**0.9536**	**0.9536**	0.9542	**0.9536**	0.9564	**0.9772**	0.9792	0.9793	0.9793	0.9785	0.8657	**0.8513**	0.8587	0.8568	0.8645
		2	0.5957	**0.5956**	0.5957	**0.5956**	**0.5956**	**0.6381**	0.6407	0.6407	0.6390	0.6407	0.4826	**0.4822**	**0.4822**	**0.4822**	0.4995
		10	**0.1706**	**0.1706**	**0.1706**	**0.1706**	0.1788	**0.2099**	**0.2099**	**0.2099**	**0.2099**	0.2167	**0.1423**	**0.1423**	0.1468	0.1458	0.1463
		25	0.0843	0.0843	**0.0841**	**0.0841**	0.0845	0.1078	0.1078	**0.1074**	0.1078	**0.1074**	**0.0649**	**0.0649**	**0.0649**	**0.0649**	0.0651
	1000	0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9955**	**0.9955**	**0.9955**	**0.9955**	0.9956
		0.5	0.9733	**0.9731**	0.9735	0.9733	0.9740	0.9629	**0.9628**	**0.9628**	0.9629	0.9632	**0.8277**	**0.8277**	**0.8277**	**0.8277**	0.8289
		2	0.6417	**0.6415**	0.6418	0.6417	0.6421	**0.6498**	0.6499	0.6499	0.6499	0.6499	**0.4589**	**0.4589**	**0.4589**	0.4590	0.4596
		10	**0.1908**	**0.1908**	**0.1908**	**0.1908**	0.1911	**0.2175**	**0.2175**	**0.2175**	**0.2175**	**0.2175**	0.1574	0.1574	**0.1573**	0.1574	0.1574
		25	**0.0852**	**0.0852**	**0.0852**	**0.0852**	0.0854	**0.0941**	**0.0941**	0.0942	**0.0941**	0.0942	**0.0702**	**0.0702**	**0.0702**	**0.0702**	0.0705
3	30	0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9689	0.9974	0.9960	**0.9649**	0.9980
		0.5	0.9716	0.9716	0.9716	0.9814	**0.9548**	0.9588	0.9588	**0.9413**	0.9588	0.9588	**0.8319**	0.8436	0.8650	0.8650	0.8766
		2	0.5970	0.5970	0.6145	0.5970	**0.5969**	**0.5949**	0.6046	0.6046	**0.5949**	**0.5949**	**0.4762**	**0.4762**	0.4798	0.4798	0.5486
		10	0.1093	0.1061	0.1061	**0.1018**	0.1093	0.1409	0.1409	**0.1360**	**0.1360**	**0.1360**	**0.0963**	0.0998	0.0998	**0.0963**	0.1022
		25	0.0373	**0.0366**	0.0373	0.0370	0.0390	**0.0505**	0.0513	0.0513	**0.0505**	0.0543	0.0393	0.0393	**0.0346**	0.0393	0.0408
	100	0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9974**	0.9986	0.9987	0.9986	0.9986
		0.5	**0.9582**	**0.9582**	**0.9582**	**0.9582**	**0.9582**	0.9884	0.9881	0.9852	0.9884	**0.9851**	0.8798	**0.8795**	0.8798	**0.8795**	0.8888
		2	0.5287	0.5187	0.5187	**0.5148**	0.5216	**0.6065**	**0.6065**	**0.6065**	**0.6065**	0.6180	**0.4614**	0.4623	0.4685	0.5074	0.4975
		10	**0.0975**	**0.0975**	**0.0975**	**0.0975**	0.1013	0.1176	0.1176	0.1176	**0.1171**	0.1175	0.0894	0.0873	**0.0872**	0.0893	0.0897
		25	**0.0353**	0.0358	**0.0353**	0.0354	0.0381	0.0408	**0.0407**	**0.0407**	0.0408	0.0423	0.0280	0.0281	**0.0279**	0.0281	0.0317
	1000	0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9993**	0.9994	**0.9993**	**0.9993**	0.9994
		0.5	0.9868	**0.9867**	0.9868	0.9868	0.9872	0.9807	0.9807	**0.9806**	**0.9806**	0.9811	0.8800	0.8796	**0.8790**	0.8797	0.8929
		2	**0.5975**	0.5976	**0.5975**	0.5976	0.5977	**0.6440**	**0.6440**	**0.6440**	**0.6440**	0.6453	**0.4614**	**0.4614**	**0.4614**	**0.4614**	0.4652
		10	**0.1124**	**0.1124**	**0.1124**	**0.1124**	0.1125	0.1169	**0.1168**	0.1169	0.1169	0.1170	**0.0837**	**0.0837**	**0.0837**	**0.0837**	0.0838
		25	**0.0333**	0.0334	**0.0333**	**0.0333**	0.0336	**0.0363**	**0.0363**	**0.0363**	**0.0363**	**0.0363**	**0.0239**	**0.0239**	**0.0239**	**0.0239**	0.0241
5	30	0.1	0.9998	0.9997	**0.9994**	0.9998	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	0.9942	0.9963	0.9988	**0.9920**	0.9981
		0.5	**0.8296**	0.8620	0.8606	0.8964	0.9254	0.9477	0.9590	**0.9456**	0.9520	**0.9456**	0.9169	0.9390	**0.8980**	0.9116	0.9477
		2	0.3394	0.3325	0.3286	**0.3266**	0.3550	0.4832	0.4554	0.4845	**0.4242**	0.4637	0.3844	0.4014	0.3917	0.4336	**0.3802**
		10	0.0200	**0.0189**	**0.0189**	0.0194	0.0237	0.0472	0.0445	**0.0425**	0.0466	0.0445	0.0451	**0.0418**	**0.0418**	**0.0418**	**0.0418**
		25	0.0044	**0.0036**	0.0038	0.0044	0.0041	**0.0072**	**0.0072**	**0.0072**	**0.0072**	**0.0072**	0.0059	0.0058	0.0059	**0.0055**	0.0058

	100	0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9980**	0.9981	0.9983	0.9984	0.9992
		0.5	0.9866	0.9866	0.9866	0.9866	**0.9852**	**0.9753**	**0.9753**	**0.9753**	**0.9753**	0.9816	0.9212	0.9349	**0.9160**	0.9212	0.9544
		2	0.4605	0.4605	0.4666	**0.4589**	0.4669	0.4375	0.4308	**0.4255**	0.4505	0.4499	**0.3264**	0.3433	0.3434	0.3391	0.3496
		10	0.0328	0.0324	0.0327	**0.0323**	0.0329	0.0310	0.0311	**0.0299**	0.0306	0.0329	0.0174	0.0174	**0.0170**	0.0180	0.0172
		25	**0.0044**	**0.0044**	**0.0044**	**0.0044**	0.0045	0.0055	0.0055	**0.0054**	0.0055	0.0055	**0.0025**	**0.0025**	0.0026	0.0027	0.0027
	1000	0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		0.5	**0.9958**	0.9960	0.9959	0.9959	0.9961	**0.9942**	**0.9942**	**0.9942**	**0.9942**	0.9943	0.9384	0.9391	**0.9377**	0.9386	0.9420
		2	**0.5269**	0.5270	**0.5269**	0.5271	0.5271	0.5587	**0.5585**	0.5586	0.5586	0.5590	0.3964	0.3960	0.3961	**0.3959**	0.3997
		10	**0.0307**	**0.0307**	**0.0307**	**0.0307**	**0.0307**	0.0406	0.0406	**0.0405**	0.0406	0.0406	**0.0249**	**0.0249**	**0.0249**	**0.0249**	0.0250
		25	**0.0043**	**0.0043**	**0.0043**	**0.0043**	**0.0043**	**0.0054**	**0.0054**	**0.0054**	**0.0054**	**0.0054**	**0.0032**	**0.0032**	**0.0032**	**0.0032**	**0.0032**

Table 7. *pswamp* values for outlier scenarios 2.

p	n	δ	$\varepsilon = 5\%$					$\varepsilon = 15\%$					$\varepsilon = 25\%$				
			FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC	FMCD	MVV	CME	ISE	TOC
2	30	0.1	**0.1837**	0.1982	**0.1837**	**0.1837**	0.1982	0.2030	**0.1914**	0.2090	0.1943	0.1943	0.4928	**0.4476**	0.4928	0.4530	0.4530
		0.5	**0.1376**	0.1494	**0.1376**	**0.1376**	0.1494	0.1832	0.1832	0.1832	0.1832	**0.1695**	0.2929	0.2929	**0.2888**	0.3040	0.2929
		2	**0.1101**	**0.1101**	**0.1101**	0.1113	**0.1101**	**0.1535**	**0.1535**	0.1542	**0.1535**	0.1572	**0.1606**	0.1761	0.1839	0.1668	0.1761
		10	**0.1018**	**0.1018**	**0.1018**	**0.1018**	**0.1018**	0.1231	**0.1110**	**0.1110**	0.1289	0.1258	**0.1535**	**0.1535**	**0.1535**	**0.1535**	**0.1535**
		25	**0.0880**	0.0896	**0.0880**	0.0946	0.0896	0.1068	0.1089	0.1068	0.1068	**0.0860**	0.1459	**0.1029**	0.1124	0.1124	0.1125
	100	0.1	0.2376	0.2387	**0.2326**	**0.2326**	0.2376	0.2601	0.2601	0.2601	0.2601	**0.2488**	0.5609	0.5426	0.5609	0.5609	**0.5396**
		0.5	0.2018	0.2018	0.2007	0.2018	**0.1980**	0.1452	**0.1413**	**0.1413**	**0.1413**	0.1425	**0.3580**	0.3694	0.3637	0.3654	0.3590
		2	**0.1694**	0.1701	**0.1694**	0.1701	0.1701	0.1388	**0.1358**	**0.1358**	0.1382	**0.1358**	0.2787	0.2776	0.2776	0.2776	**0.2566**
		10	0.1658	0.1658	0.1658	0.1658	**0.1450**	0.1031	0.1031	0.1031	0.1031	**0.0896**	0.2517	0.2425	0.2380	0.2106	**0.2090**
		25	**0.1174**	0.1175	**0.1174**	**0.1174**	0.1271	**0.0621**	**0.0621**	0.0651	**0.0621**	0.0651	0.2078	0.2078	0.2078	0.2078	**0.1997**
	1000	0.1	0.1669	0.1669	0.1669	0.1669	**0.1636**	0.2195	0.2194	0.2195	0.2198	**0.2193**	0.5797	0.5797	0.5797	0.5797	**0.5790**
		0.5	0.1586	0.1589	0.1583	0.1586	**0.1574**	0.1919	0.1922	0.1921	0.1920	**0.1915**	0.4132	0.4132	0.4132	0.4132	**0.4122**
		2	0.1294	0.1296	0.1294	0.1294	**0.1291**	**0.1237**	**0.1237**	**0.1237**	**0.1237**	0.1238	0.2927	0.2927	0.2926	0.2924	**0.2915**
		10	0.1241	0.1241	0.1241	0.1241	**0.1232**	**0.0869**	**0.0869**	**0.0869**	**0.0869**	0.0870	0.1810	0.1811	0.1808	**0.1807**	0.1810
		25	0.1104	0.1103	0.1104	0.1105	**0.1086**	**0.0850**	**0.0850**	0.0851	0.0852	**0.0850**	0.1675	0.1672	0.1675	0.1678	**0.1626**
3	30	0.1	0.2124	0.2124	0.2373	0.2124	**0.2106**	0.2864	0.2513	0.2513	0.2663	**0.2485**	0.5992	0.5342	0.5571	0.6413	**0.5169**
		0.5	0.1827	0.1827	0.1827	**0.1592**	0.2068	**0.2129**	**0.2129**	0.2382	**0.2129**	**0.2129**	0.4342	0.4269	0.3947	0.3947	**0.3748**
		2	0.1381	0.1381	**0.1264**	0.1381	0.1481	0.1506	**0.1397**	**0.1397**	0.1506	0.1506	0.2702	0.2702	0.2684	0.2684	**0.2057**
		10	0.1339	**0.1261**	**0.1261**	0.1378	0.1339	**0.0921**	**0.0921**	0.0980	0.0980	0.0980	0.1732	**0.1527**	**0.1527**	0.1732	0.1913
		25	**0.1015**	0.1109	**0.1015**	0.1104	0.1213	0.0529	0.0522	0.0522	0.0529	**0.0405**	**0.1186**	**0.1186**	0.1519	**0.1186**	0.1210

	100	0.1	**0.2103**	0.2166	0.2122	**0.2103**	0.2194	0.1844	0.1844	0.1844	0.1852	**0.1766**	0.6045	0.5908	**0.5892**	0.5908	0.5908
		0.5	**0.2101**	**0.2101**	**0.2101**	**0.2101**	**0.2101**	**0.1293**	0.1305	0.1376	**0.1293**	0.1389	0.3485	0.3496	0.3485	0.3496	**0.3384**
		2	**0.1808**	0.1958	0.1958	0.1998	0.1915	0.1238	0.1238	0.1238	0.1238	**0.1115**	0.2565	0.2559	0.2543	**0.2043**	0.2158
		10	0.1331	0.1331	0.1331	0.1331	**0.1226**	**0.1038**	0.1043	**0.1038**	0.1065	0.1071	0.1697	0.1921	0.1946	0.1980	**0.1682**
		25	0.1042	0.0890	0.1042	0.1037	**0.0707**	0.0624	0.0660	0.0660	0.0624	**0.0541**	0.1362	0.1363	0.1301	0.1363	**0.1179**
	1000	0.1	0.1563	0.1573	0.1573	0.1563	**0.1557**	0.2132	0.2130	0.2130	0.2130	**0.2120**	0.6179	0.6179	0.6182	0.6179	**0.6170**
		0.5	0.1423	0.1425	0.1419	0.1419	**0.1407**	0.1731	0.1731	0.1733	0.1734	**0.1723**	0.3893	0.3898	0.3904	0.3897	**0.3776**
		2	0.1209	**0.1207**	0.1208	0.1208	**0.1207**	0.0910	0.0910	0.0910	0.0910	**0.0898**	0.2313	0.2313	0.2313	0.2313	**0.2253**
		10	0.0965	0.0966	0.0965	0.0965	**0.0961**	0.0887	0.0888	0.0886	0.0886	**0.0882**	0.1691	0.1687	0.1687	0.1691	**0.1683**
		25	0.0901	0.0901	0.0899	0.0900	**0.0874**	0.0730	0.0730	0.0729	0.0730	**0.0728**	0.1593	0.1591	0.1593	0.1593	**0.1533**
5	30	0.1	**0.4689**	0.4736	0.5365	0.4844	0.4981	0.3463	0.3658	0.3329	**0.3208**	0.3392	0.7052	0.6759	**0.6399**	0.7044	0.6784
		0.5	0.4342	0.4165	0.4078	0.3644	**0.3292**	0.2663	**0.2395**	0.2635	0.2639	0.2635	0.3269	0.2858	0.3484	0.3322	**0.2747**
		2	0.3189	0.3251	0.3358	0.3264	**0.2831**	0.1897	0.2041	**0.1846**	0.2363	0.2055	0.2731	0.2539	0.2750	**0.2238**	0.2722
		10	0.2456	0.2735	0.2735	0.2747	**0.2369**	0.1175	**0.1090**	0.1370	0.1395	**0.1090**	**0.1189**	0.1218	0.1218	0.1218	0.1218
		25	0.1891	0.1670	**0.1634**	0.1816	0.1700	**0.0748**	**0.0748**	**0.0748**	**0.0748**	**0.0748**	0.1126	0.1188	0.1126	**0.0823**	0.1188
	100	0.1	**0.1922**	0.1966	0.1987	0.1937	0.1932	0.2481	**0.2381**	0.2423	0.2488	0.2481	0.7286	0.7272	0.7242	0.7216	**0.7129**
		0.5	**0.1718**	**0.1718**	**0.1718**	**0.1718**	0.1781	0.2197	0.2197	0.2197	0.2197	**0.2011**	0.3696	0.3464	0.3693	0.3696	**0.3144**
		2	0.1621	0.1621	0.1570	0.1651	**0.1561**	0.1940	0.2010	0.2073	0.1802	**0.1790**	0.3142	0.2847	0.2779	0.2863	**0.2696**
		10	0.1226	0.1344	0.1344	0.1302	**0.1203**	0.1382	0.1489	0.1404	0.1421	**0.1081**	0.2571	0.2577	0.2670	**0.2442**	0.2594
		25	0.1036	0.1036	0.1036	0.1036	**0.0941**	**0.0710**	0.0713	0.0750	0.0713	0.0713	0.2428	0.2338	0.2166	**0.2014**	0.2207
	1000	0.1	0.1351	0.1355	0.1351	0.1349	**0.1341**	0.2215	0.2215	0.2214	0.2215	**0.2206**	0.6885	0.6891	0.6885	0.6884	**0.6871**
		0.5	0.1224	0.1216	0.1220	0.1218	**0.1203**	0.1424	0.1424	0.1424	0.1424	**0.1417**	0.3744	0.3736	0.3748	0.3740	**0.3688**
		2	0.1088	**0.1085**	0.1088	**0.1085**	0.1086	0.0894	0.0897	0.0896	0.0895	**0.0892**	0.2063	0.2072	0.2068	0.2072	**0.2018**
		10	0.1077	0.1075	0.1076	0.1076	**0.1072**	0.0678	0.0677	0.0675	0.0677	**0.0668**	0.1463	0.1459	0.1450	0.1456	**0.1440**
		25	0.0891	0.0893	0.0889	0.0890	**0.0879**	0.0639	0.0639	0.0638	0.0640	**0.0637**	0.1384	0.1373	0.1382	0.1379	**0.1363**

Table 7 shows the probability values ofall robust estimators misclassify inliers as outliers, *pswamp* for 5%, 15% and 25% percentage of outliers. Results also show *pswamp* values decrease as the values δ increase. It shows that as the distance between outliers and inliers increases, the performance of robust estimators is better.

The TOC gives better results for most of the cases of 5%, 15% and 25% outliers. It is found that for 5% outliers, TOC gives better results than other robust estimators for 24 out of 45 cases. ISE gives better results than other robust estimators for 1 out of 45 cases. While FMCD and CME give better performance with other robust estimators for 3 and 2 out of 45 cases. For equal performance to not misclassify inliers as outliers, CME and FMCD show 12and 11 out of 45 cases has the same results with other robust estimators. Meanwhile, MVV, ISE and TOC give an equal performance with other robust estimators for 7, 9 and 4 out of 45 cases, respectively.

For 15% outliers, 24 out of 45 cases show TOC has the lowest value of *pswamp*. FMCD, CME, and ISE show that 1 out of 45 cases give better results than other robust estimators. While MVV gives better results than other robust estimators for 3 out of 45 cases. For equal performance to not misclassify inliers as outliers, CME and ISE show 8 out of 45 cases have the same results with other robust estimators. FMCD, MVV and TOC each give an equal performance with other robust estimators for 10, 13 and 5 out of 45 cases, respectively.

While for 25% outliers 28 out of 45 cases show TOC has the lowest value of *pswamp*. FMCD and CME show 3 out of 45 cases give better results than other robust estimators. While MVV and ISE show 2 and 6 out of 45 cases give better results than other robust estimators. Results for equal performance show MVV and TOC has 3and 1 out of 45 cases give same results with other robust estimators. Meanwhile FMCD, CME and ISE show 2 out of 45 cases give an equal performance with other robust estimators.

3.3 The Best Robust Estimator

Tables 8 – 10 show summary results for *pout*, *pmask* and *pswamp* for both outlier scenarios. From Table 8, all robust estimators give the best result for outlier scenario 1 when $\lambda = 4$ and $\lambda = 10$. For outlier scenario 2 most of the robust estimators give better results when $\delta = 10$ and $\delta = 25$. The same results were also obtained for *pmask* where most of the robust estimators give the best results when the distance of outliers and inliers are large for both outlier scenarios. From Table 10, TOC can be said to be the best estimator for *pswamp*. Both outlier scenarios show TOC is the best estimator for most cases.

Table 8. The best robust estimators for *pout* using Outlier Scenarios 1 and Outlier Scenarios 2.

ε	n	p	Outlier Scenarios 1				Outlier Scenarios 2				
			$\lambda=1$	$\lambda=2$	$\lambda=4$	$\lambda=10$	$\delta=0.1$	$\delta=0.5$	$\delta=2$	$\delta=10$	$\delta=25$
5%	30	2	All	FMCD, CME	All	All	None	MVV, TOC	ISE	All	ISE
		3	FMCD	FMCD	All	All	None	TOC	FMCD, MVV, ISE	ISE	MVV
		5	ISE	MVV	All	All	None	FMCD	ISE	MVV, CME	MVV
	100	2	TOC	TOC	FMCD, MVV,CME	All	None	None	MVV, ISE, TOC	FMCD, MVV, CME, ISE	FMCD, CME, ISE
		3	MVV, CME	All	All	All	None	None	ISE	FMCD, MVV, CME, ISE	ISE
		5	MVV	MVV	All	All	None	None	ISE	ISE	FMCD, MVV, CME, ISE
	1000	2	None	None	FMCD, MVV, CME, ISE	All	None	None	None	None	FMCD, CME, ISE
		3	None	FMCD, MVV, CME	All	All	None	None	None	MVV	FMCD
		5	None	ISE	All	All	None	None	None	FMCD, MVV	FMCD
15%	30	2	CME	All	MVV, CME	All	None	None	FMCD, MVV, ISE	ISE	MVV
		3	ISE, MVV	FMCD, CME, TOC	All	All	None	None	FMCD, ISE, TOC	CME, ISE, TOC	FMCD, ISE
		5	FMCD	CME	All	All	None	None	ISE	CME	All
	100	2	None	FMCD, MVV, TOC	All	All	None	None	None	FMCD, MVV, CME, ISE	CME, TOC
		3	None	FMCD, CME, ISE, TOC	MVV	All	None	None	None	ISE	MVV, CME
		5	None	FMCD, MVV, ISE	All	All	None	None	None	CME	CME
	1000	2	None	None	All	All	None	None	None	None	None
		3	None	None	All	All	None	None	None	None	FMCD, MVV, CME, ISE
		5	None	CME	All	All	None	None	None	FMCD	FMCD, MVV
25%	30	2	MVV, CME	FMCD, ISE	All	All	None	None	FMCD	All	FMCD
		3	MVV	ISE	FMCD, MVV, CME, ISE	All	None	None	FMCD, MVV	FMCD, ISE	CME
		5	ISE	TOC	CME	MVV, CME, ISE	None	None	TOC	MVV, CME, ISE, TOC	ISE
	100	2	None	MVV, CME	FMCD, MVV, CME, ISE	All	None	None	None	FMCD	FMCD, MVV, CME, ISE
		3	None	CME	All	All	None	None	None	CME	CME
		5	None	FMCD, TOC	All	All	None	CME	None	CME	FMCD
	1000	2	None	None	FMCD, MVV, CME, ISE	All	None	None	None	None	None
		3	None	None	All	All	None	None	None	None	FMCD, CME, ISE
		5	None	FMCD, CME	All	All	None	None	None	ISE, TOC	ISE

Table 9. The best robust estimators for *pmask* using Outlier Scenarios 1 and Outlier Scenarios 2.

ε	n	p	Outlier Scenarios 1				Outlier Scenarios 2				
			$\lambda=1$	$\lambda=2$	$\lambda=4$	$\lambda=10$	$\delta=0.1$	$\delta=0.5$	$\delta=2$	$\delta=10$	$\delta=25$
5%	30	2	All	FMCD, CME	All	All	None	MVV, TOC	ISE	All	ISE
		3	FMCD	FMCD	All	All	None	TOC	TOC	ISE	MVV
		5	ISE	MVV	All	All	CME	FMCD	ISE	MVV, CME	MVV
	100	2	TOC	TOC	All	All	None	FMCD, MVV, ISE	MVV, ISE, TOC	FMCD, MVV, CME, ISE	CME, ISE
		3	MVV, CME	All	All	All	None	All	ISE	FMCD, MVV, CME, ISE	FMCD, CME
		5	MVV	MVV	All	All	None	TOC	ISE	ISE	FMCD, MVV, CME, ISE
	1000	2	TOC	FMCD, MVV, CME, ISE	All	All	None	MVV	MVV	FMCD, MVV, CME, ISE	FMCD, MVV, CME, ISE
		3	TOC	FMCD, MVV, CME	All	All	None	MVV	FMCD, CME	FMCD, MVV, CME, ISE	FMCD, CME, ISE
		5	MVV	ISE	All	All	None	FMCD	FMCD, CME	All	All
15%	30	2	CME	All	MVV, CME, TOC	All	None	FMCD, MVV, CME, ISE	CME	ISE	MVV
		3	MVV, ISE, TOC	FMCD, CME	All	All	None	CME	FMCD, ISE, TOC	CME, ISE, TOC	FMCD, ISE
		5	FMCD	CME	All	All	None	CME, TOC	ISE	CME	All
	100	2	FMCD, MVV, CME, ISE	FMCD, MVV	TOC	All	None	FMCD	FMCD	FMCD, MVV, CME, ISE	CME, TOC
		3	FMCD, MVV, CME	FMCD, CME, ISE	TOC	All	None	TOC	FMCD, MVV, CME, ISE	ISE	MVV, CME
		5	FMCD	FMCD, MVV, ISE	All	All	None	FMCD, MVV, CME, ISE	CME	CME	CME
	1000	2	FMCD, MVV, CME, ISE	FMCD, MVV, CME, ISE	All	All	None	MVV, CME	FMCD	All	FMCD, MVV, ISE
		3	MVV	FMCD, MVV, CME, ISE	All	All	None	CME, ISE	FMCD, MVV, CME, ISE	MVV	All
		5	CME	CME	All	All	None	FMCD, MVV, CME, ISE	MVV	CME	All
25%	30	2	MVV, CME	FMCD, ISE	All	All	FMCD, CME	ISE	MVV, TOC	All	FMCD
		3	FMCD, TOC	ISE	All	All	ISE	FMCD	FMCD, MVV	FMCD, ISE	CME
		5	FMCD	TOC	CME	MVV, CME, ISE	ISE	CME	TOC	MVV, CME, ISE, TOC	ISE
	100	2	MVV	MVV, CME	FMCD, MVV, CME, ISE	All	FMCD, CME, ISE	MVV	MVV, CME, ISE	FMCD, MVV	FMCD, MVV, CME, ISE
		3	ISE	CME	All	All	FMCD	MVV, ISE	FMCD	CME	CME
		5	CME	FMCD, TOC	All	All	FMCD	CME	FMCD	CME	FMCD, MVV
	1000	2	FMCD, MVV, CME, ISE	FMCD, MVV, CME, ISE	FMCD, MVV, CME, ISE	All	FMCD, MVV, CME, ISE	FMCD, MVV, CME, ISE	FMCD, MVV, CME	CME	FMCD, MVV, CME, ISE
		3	CME	FMCD, MVV, ISE	All	All	FMCD, CME, ISE	CME	FMCD, MVV, CME, ISE	FMCD, MVV, CME, ISE	FMCD, MVV, CME, ISE
		5	CME	FMCD, CME	All	All	None	CME	ISE	FMCD, MVV, CME, ISE	All

Table 10. The best robust estimators for *pswamp* using Outlier Scenarios 1 and Outlier Scenarios 2.

ε	n	p	Outlier Scenarios 1				Outlier Scenarios 2				
			$\lambda=1$	$\lambda=2$	$\lambda=4$	$\lambda=10$	$\delta=0.1$	$\delta=0.5$	$\delta=2$	$\delta=10$	$\delta=25$
5%	30	2	All	TOC	MVV	All	FMCD, CME, ISE	FMCD, CME, ISE	FMCD, MVV, CME, TOC	All	FMCD, CME
		3	MVV	TOC	TOC	TOC	TOC	ISE	CME	MVV, CME	FMCD, CME
		5	ISE	TOC	MVV, TOC	TOC	FMCD	TOC	TOC	TOC	CME
	100	2	TOC	TOC	TOC	TOC	CME, ISE	TOC	FMCD, CME	TOC	FMCD, CME, ISE
		3	FMCD	All	FMCD, MVV, ISE, TOC	TOC	FMCD, ISE	All	FMCD	TOC	TOC
		5	ISE	TOC	TOC	MVV	FMCD	FMCD, MVV, CME, ISE	TOC	TOC	TOC
	1000	2	TOC	FMCD, MVV, CME, ISE	FMCD, MVV, CME, ISE	TOC	TOC	TOC	TOC	TOC	TOC
		3	TOC	TOC	TOC	ISE	TOC	TOC	MVV, TOC	TOC	TOC
		5	TOC	TOC	All	TOC	TOC	TOC	MVV, ISE	TOC	TOC
15%	30	2	FMCD	All	MVV, CME, TOC	FMCD	MVV	TOC	FMCD, MVV, ISE	MVV, CME	TOC
		3	CME	FMCD, CME	FMCD, ISE, TOC	All	TOC	FMCD, MVV, ISE, TOC	MVV, CME	FMCD, MVV	TOC
		5	MVV	MVV	MVV, ISE	TOC	ISE	MVV	CME	MVV, TOC	All
	100	2	TOC	ISE	TOC	All	TOC	MVV, CME, ISE	MVV, CME, TOC	TOC	FMCD, MVV, ISE
		3	TOC	TOC	TOC	TOC	TOC	FMCD, ISE	TOC	FMCD, CME	TOC
		5	TOC	CME	FMCD, CME, ISE	MVV	MVV	TOC	TOC	TOC	FMCD
	1000	2	TOC	FMCD, MVV, CME, ISE	TOC	FMCD, MVV, CME, ISE	TOC	TOC	FMCD, MVV, CME, ISE	FMCD, MVV, CME, ISE	FMCD, MVV, TOC
		3	TOC	FMCD, MVV, CME, ISE	TOC	TOC	TOC	TOC	TOC	TOC	TOC
		5	TOC	All	TOC	TOC	TOC	TOC	TOC	TOC	TOC
25%	30	2	MVV, CME	MVV	All	All	MVV	CME	FMCD	All	MVV
		3	FMCD, TOC	TOC	MVV, CME, ISE	All	TOC	TOC	TOC	MVV, CME	FMCD, MVV, ISE
		5	ISE	MVV	CME	FMCD	CME	TOC	ISE	FMCD	ISE
	100	2	TOC	FMCD	TOC	TOC	TOC	FMCD	TOC	TOC	TOC
		3	CME, TOC	CME	FMCD	ISE	CME	TOC	ISE	TOC	TOC
		5	TOC	FMCD, TOC	ISE	MVV, ISE	TOC	TOC	TOC	ISE	ISE
	1000	2	FMCD, MVV, CME, ISE	FMCD, MVV, CME, ISE	TOC	FMCD, MVV, CME, ISE	TOC	TOC	TOC	ISE	TOC
		3	TOC	FMCD, MVV, CME, ISE	TOC	TOC	TOC	TOC	TOC	TOC	TOC
		5	TOC	MVV, ISE	TOC	TOC	TOC	TOC	TOC	TOC	TOC

5. Conclusion

In this study, the performance of five robust estimators which are FMCD, MVV, CME, ISE and TOC are compared. All the robust estimators are the innovation method from FMCD. This study is done to investigate further the performance of the latest robust estimator namely TOC for various sample sizes, number of variables, percentage of outliers and two different outlier scenarios. From the simulation study, results indicate that TOC is considered good when the distance of outliers increase even though TOC has the highest value of *pmask* and most of the value of *pswamp* are the lowest. This simulation study has also shown that the use of TOC is applicable and a promising approach to detect outliers for multivariate data especially when the distance between outliers and inliers is large for sample size, $n = 30, 100, 1000$ and the number of variables, $p = 2, 3, 5$. Besides that, TOC shows the lowest values of *pswamp* for most of the cases.

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7. References

Andrea, C., Marco, R. & Francesca, T. (2011). Accurate and powerful multivariate outlier detection. In *Proceedings 58th World Congress, 2011, Dublin* (pp. 5608 - 5613).

Abd Mutalib SSS, Satari SZ, Wan Yusoff WNS. (2019). A New Robust Estimator to Detect Outliers for Multivariate Data. In: *Journal of Physics: Conference Series, 1366,* 1 – 9.

Cabana, E., Henry, L. & Lillo, R. E. (2017). *Multivariate outlier detection based on a robust Mahalanobis distance with shrinkage estimators*, (DES - Working Papers. Statistics and Econometrics. WS 24613), Universidad Carlos III de Madrid. Departamento de Estadística. https://e-archivo.uc3m.es/bitstream/handle/10016/24613/ws201710.pdf.

Cerioli, A. (2010). Multivariate outlier detection with high breakdown estimators. *Journal of the American Statistical Association*, *105*(489), 147 - 156.

Djauhari, M. A. (2008). Highly robust estimation of location and scatter when data sets are of high dimension: An open problem. In *The 3rd International Conference on Mathematics and Statistics (ICoMS-3)* (pp. 1–8). Institut Pertanian Bogor, Indonesia.

Fauconnier, C. & Haesbroeck, G. (2009). Outliers detection with the minimum covariance determinant estimator in practice. *Statistical Methodologyethodology,* 363 - 379.

Filzmoser, P., & Todorov, V. (2013). Robust tools for the imperfect world. *Information Sciences*, *245*, 4–20.

Filzmoser, P. (2005). Identification of multivariate outliers: A performance study. *Austrian Journal of Statistics*, *34*(2), 127–138.

Filzmoser, P., Maronna, R. & Werner, M. (2008). Outlier identification in high dimensions. *Computational Statistics & Data Analysis, 52*(3), 1694 - 1711.

Hubert, Mia, & Debruyne, M. (2010). Minimum covariance determinant. *Wiley Interdisciplinary Reviews: Computational Statistics*, *2*(1), 36–43.

Hadi, A. S., Rahmatullah Imon, A. H. M., & Werner, M. (2009). Detection of outliers. *Wiley Interdisciplinary Reviews: Computational Statistics*, *1*, 57–70.

Herwindiati, D. E., Djauhari, M. A., & Mashuri, M. (2007). Robust multivariate outlier labeling. *Communications in Statistics: Simulation and Computation*, *36*, 1287–1294.

Kosinski, A. S. (1999). A procedure for the detection of multivariate outliers. *Computational Statistics & Data Analysis, 29,* 145 - 161.

Leys, C., Klein, O., Dominicy, Y., & Ley, C. (2018). Detecting multivariate outliers: Use a robust variant of the Mahalanobis distance. *Journal f Experimental Social Psychology, 74,* 150 - 156.

Lim, H. A., & Midi, H. (2016). Diagnostic Robust Generalized Potential based on Index Set Equality (DRGP (ISE)) for the identification of high leverage points in linear model. *Computational Statistics*, *31*, 859–877.

Möller, S. F., Frese, J. Von, & Bro, R. (2005). Robust Methods for Multivarite Data Analysis. *Journal of Chemometrics*, *19*, 549–563.

Pan, J-X., Fung, W-K. & Fang, K-T. (2000). Multiple outlier detection multivariate data using projection pursuit techniques. *Journal of Statistical Planning,* 83, 153-167.

Rencher, A. C. (2002). *Methods of multivariate analysis*. *Wiley-Interscience*. John Wiley and Sons Ltd.

Rocke, D. M. & Woodruff, D. L. (1996). Identification of outliers in multivariate data. *Journal of the American Statistical Association, 91*(435), 1047 - 1061.

Rohayu, M. S. (2013). *A robust estimation method of location and scale with application in monitoring process variability* [Unpublished Doctoral thesis]. Universiti Teknologi Malaysia.

Rousseeuw, P. J., & Van Driessen, K. (1999). A fast algorithm for the Minimum Covariance Determinant estimator. *Technometrics*, *41*(3), 212–223.

Rousseeuw, P. J. (1984). Least median of squares regression. *Journal of the American Statistical Association*, *79* (388), 871–880.

Rousseeuw P., Yohai V. (1984). Robust Regression by Means of S-Estimators. In: Franke J., Härdle W., Martin D. (Eds). *Robust and Nonlinear Time Series Analysis*, 26: 256-272. Lecture Notes in Statistics, Springer, New York, NY.

Rousseeuw, Peter. (1985). Multivariate estimation with high breakdown point. *Mathematical Statistics* *and Applications*, 283–297.

Ro, K., Zou, C., Wang, Z., & Yin, G. (2015). Outlier detection for high-dimensional data. *Biometrika*, *102*(3), 589–599.

Salleh, R. M., & Djauhari, M. A. (2011). Robust hotelling's T^2 control charting in spike production process. In *International Seminar on the Application of Science & Mathematics 2011 (ISASM 2011)* (pp. 1–8).

Santos-Pereira C.M., Pires A.M. (2013). Robust Clustering Method for the Detection of Outliers: Using AIC to Select the Number of Clusters. In: Lita da Silva J., Caeiro F., Natário I., Braumann C. (Eds). *Advances in Regression, Survival Analysis, Extreme Values, Markov Processes and Other Statistical Applications* (pp. 409-415). Studies in Theoretical and Applied Statistics. Springer, Berlin, Heidelberg.

Su, X., & Tsai, C.-L. (2011). Outlier detection. *Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery*, *1*, 261–268.

Sebert, D. M., Montgomery, D. C., & Rollier, D. A. (1998). A clustering algorithm for identifying multiple outliers in linear regression. *Computational Statistics and Data Analysis*, *27*(4), 461–484.

Santos-pereira, C. M., & Pires, A. M. (2002). Detection of outliers in multivariate data: A method based on clustering and robust estimators.

Werner, M. (2003). *Identification of multivariate outliers in large data sets* (Doctoral thesis, University of Colorado, Denver). http://math.ucdenver.edu/graduate/thesis/werner_thesis.pdf.