

RELIABILITY ANALYSIS TO INDUSTRIAL ACTIVE STANDBY REDUNDANT SYSTEM

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ABSTRACT The present paper is contributed for the analysis of system possessing cold standby i.e. active standby redundancy mode. Initially, both similar units are observed to be in operative situation. The inspection procedure for failed units has been introduced who inspect the exact failure and communicate the repairman to repair the exact failed part of unit by replacing with new one with same performance capacity. As the system is following discrete mechanism so the results related to system reliability like MTSF, system availability in operative state, repair & failed time are obtained with application of geometric distribution & regenerative technique. Graphical analysis had also been workout for analyzing the behaviour of profit function with increasing/decreasing rate of repair mechanism and failure rate.

Keywords: Geometric distribution, Availability, Inspection, Busy period, Profit function.

Mathematics Subject Classification (2010): 60K10, 60K20, 90B25.

1. INTRODUCTION

Today the world is growing fast towards the direction of smart world with new ideas in each area of research and innovations. The developing and updating of these new and fast industrial techniques or machines helps in fulfilling the market requirement. As a result, the old and traditional techniques/machines which were handled manually till now are replacing with automatically ones. As these advance changes are found to be beneficiary for satisfaction of customer demands but still it leads to many complexities and taking our concern towards the reliability of industrial

equipment's. The major factor that we all know is to get into the profit or success of any new mechanism, for which we must quantify its reliability as they affect a lot to the important measures for proper utilization of system and its maintenance. In other words, we can say that the reliability is the important measure that helps our industrial system to grow by reducing the frequency of failures and minimizing the maintenance costs. As reliability is dependent upon many parameters so many researchers had contributed by analyzing many industrial and non-industrial reliability models on different technical problem with different working environment. In such system the

first thing to be taken in concern are the two categories of redundancy system i.e. hot (active) standby redundancy and cold (passive) standby redundancy. The hot (active) standby is the form of redundancy, the system has a positive probability of failure even when it is not in operation. In other words, system is said to be active redundant if all the unit's functions simultaneously and system operates even when our unit operates. The off-line unit cannot fail and is completely unloaded. Whereas, a passive redundant system is one in which one unit is operative at a time and others are kept in spare known as standby. Previous researchers had taken any one of these two categories of redundancy for analyzing many technical systems. Said et al (2005) had analyze cold standby system with random change in units and explaining the inspection procedure for deciding satisfactory repair of failed unit. In 2009, to study the behavior of dissimilar units with some common cause and multiple reason of failures and their maintenance Haggag used the concept of linear first order differential equations. Aggarwal et al (2010) had analyzed the two units under exponentially distribution for single repair time and two distinct failures. Rizwan et al. (2010) had contributed to reliability analysis by analyzing hot standby PLC system, of an industrial system. Under his study he evaluated PLC system by inspecting four failures i.e. a) failure due to corrupt software, b) power supply failure or digital relay burnt, c) complete unit burnt failure, d) input module failure. In 2013, stochastically analysis of the system with one-unit was analyzed by S. Gupta et al in which he followed the concept of post repair with inspection and with preventive maintenance/replacement. D. Singh (2014) had economically analyzed the gas turbine plant that comprised with one gas & steam turbine of an industrial system i.e. power generating system. R. Malhotra (2014) had also stochastically analyzed the system in which both units may fall in operative

condition together depending upon the system requirements. M.K. Kakkar et al had also come with new concept of analyzing industrial system following continuous distribution under correlation. S.Z. Taj, et al (2017) had initiated by studying the cable plant subsystem by framing probabilistic modeling to two identical machines operating in parallel form. The innovative idea he introduced was the evaluation of three kinds of maintenances facility (i.e. by providing repair, minor and major one) for repair of failed subsystem in seven years. The important factor to be noticed is that at failure of the subsystem it undergoes repair whereas preventive maintenance (minor & major) is performed as per schedule. In reliability field, N. Adlakha (2017) had evaluates the system by taking into the consideration of an assembling and activation time for cold standby unit to get into an operative state. Hence, under field of reliability following continuous distribution many real-life models had been designed and evaluated. But, in real world we can't predict that all system that exist are only followed by continuous distribution. In words there can be many mechanisms that comes under discrete distribution like geometric distribution. So, in these areas few researchers or innovators come with a new idea in which the reliability models must geometrically distribute for obtaining various reliability measures of the system effectiveness. From 2008 onwards N. Bhardwaj had come with this idea and stochastically analyzed the redundant system with single unit possessing with two distinct repair and failure mechanism. By keeping significance to discrete reliability modelling, Bhatti also stochastically analyzed different kinds of industrial standby units for minimizing geometric failure & repairing rates. In all past research, the important factor including filtration technique has not been studied for failed hot (active) standby redundant system. Therefore, a special initiative

taken in this paper is that, parallel systems consisting of two similar units had been analyzed by involving inspection scheme for failed machine. Initially, both similar units are observed to be in operative

situation i.e. in active standby redundancy mode. Possible states of the system under operative and failed states of the system is shown by transition model figure. 1.

Table 1. Nomenclature

Parameters	Descriptions
X_0	: Operative behaviour of unit 'X'
X_I/X_{IW}	: Inspection/waiting behaviour of failed unit 'X'.
X_R/X_{RW}	: Repairman state who repair the unit by replacing failed parts or waiting due to any reason.
p_1	: Probabilistic value of an operating unit 'X' to fall in failure mode
p_2	: Probabilistic value of a failed unit 'X' to be inspected successfully
R	: Probabilistic value of a failed unit 'X' to be repaired successfully

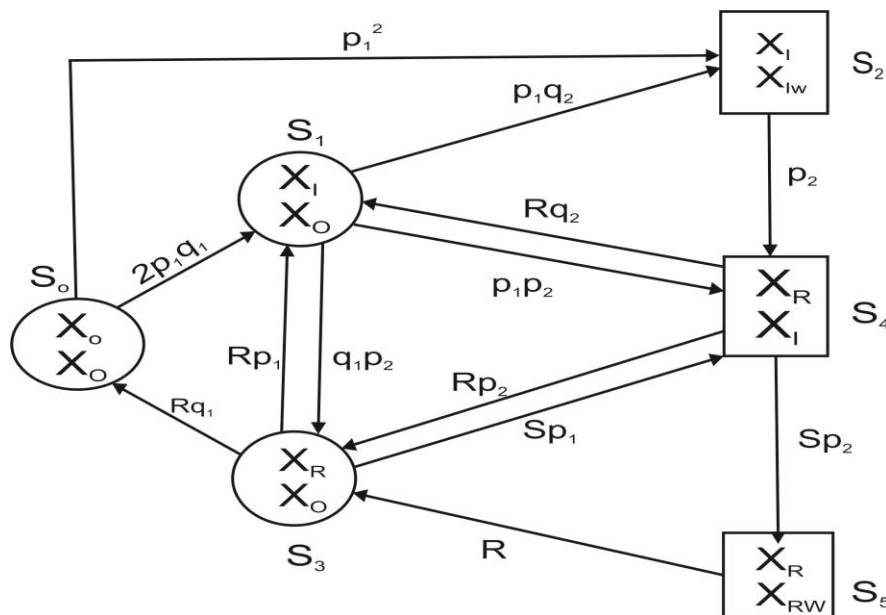


Figure 1. Transition Model

Up States	Down State
$S_0 = (X_0, X_0),$	$S_2 = (X_I, X_{IW})$
$S_1 = (X_I, X_0)$	$S_4 = (X_R, X_I)$
$S_3 = (X_R, X_0)$	$S_5 = (A_R, A_{RW})$

2. TRANSITION PROBABILITIES

Using the transition diagram shown in Figure 1 the steady state transition probabilities from state S_i to S_j can be calculated by applying:

$$P_{ij} = \lim_{t \rightarrow \infty} Q_{ij}$$

where Q_{ij} depicts the 'cumulative density function' from first regenerative state i to second j . The evaluated transition probabilities are as follows:

$$\begin{aligned} P_{01}(t) &= \frac{2p_1q_1}{1-q_1} & P_{02}(t) &= \frac{p_1^2}{1-q_1} & P_{12}(t) &= \frac{p_1q_2}{1-q_1q_2} & P_{13}(t) &= \frac{q_1p_2}{1-q_1q_2} \\ P_{14}(t) &= \frac{p_1p_2}{1-q_1q_2} & P_{24}(t) &= \frac{p_2}{1-q_2} & P_{30}(t) &= \frac{Rq_1}{1-Sq_1} & P_{31}(t) &= \frac{Rp_1}{1-Sq_1} \\ P_{34}(t) &= \frac{Sp_1}{1-Sq_1} & P_{41}(t) &= \frac{Rq_2}{1-Sq_2} & P_{43}(t) &= \frac{Rp_2}{1-Sq_2} & P_{45}(t) &= \frac{Sp_2}{1-Sq_2} \\ P_{53}(t) &= \frac{R}{1-S} \end{aligned}$$

The steady state transition probabilities from state S_i to S_j can be obtained from

$$P_{ij} = \lim_{t \rightarrow \infty} Q_{ij}$$

The above equations can be verified that

$$\begin{aligned} P_{01} + P_{02} &= 1, & P_{12} + P_{13} + P_{14} &= 1, & P_{24} &= 1, \\ P_{30} + P_{31} + P_{34} &= 1, & P_{41} + P_{43} + P_{45} &= 1, & P_{53} &= 1. \end{aligned}$$

3. MEAN SOJOURN TIMES

By mentioning sojourn time in state S_i ($i = 0 - 5$), by symbol ' μ_i ' the value of mean sojourn time for state S_i is calculated as:

$$\mu_0 = \frac{1}{1-q_1}, \quad \mu_1 = \frac{1}{1-q_1q_2}, \quad \mu_2 = \frac{1}{1-q_2}, \quad \mu_3 = \frac{1}{1-Sq_1}, \quad \mu_4 = \frac{1}{1-Sq_2}, \quad \mu_5 = \frac{1}{1-S}$$

4. MEAN TIME TO SYSTEM FAILURE (MTSF)

To calculate $MTSF$ of the proposed system, the absorbing states is taken to be the failure ones. Then, reliability analysis R_i at time ' t ' is obtained by solving the following equation:

$$\begin{aligned} R_0 &= Z_0 + q_{01} \odot R_1 \\ R_1 &= Z_1 + q_{13} \odot R_3 \\ R_3 &= Z_3 + q_{30} \odot R_0 + q_{31} \odot R_1 \end{aligned} \tag{1-3}$$

By solving above equations, we obtain

$$MTSF = \lim_{h \rightarrow 1} \frac{N_1(h)}{D_1(h)} - 1 = \frac{N_1}{D_1}$$

where

$$\begin{aligned} N_1 &= \nu_0 (1 - P_{13} P_{31}) + P_{01} (\nu_1 + \nu_3 P_{13}). \\ D_1 &= 1 - P_{13} P_{31} - P_{01} P_{13} P_{30}. \end{aligned} \tag{4-5}$$

5. AVAILABILITY ANALYSIS

Let us denote A_i to be the probability of operative/working state of system with full capacity at time 't'. Using probabilistic argument, the relations obtained are:

$$\begin{aligned} A_0 &= Z_0 + q_{01} \odot A_1 + q_{02} \odot A_2 \\ A_1 &= Z_1 + q_{12} \odot A_2 + q_{13} \odot A_3 + q_{14} \odot A_4 \\ A_2 &= q_{24} \odot A_4 \\ A_3 &= Z_3 + q_{30} \odot A_0 + q_{31} \odot A_1 + q_{34} \odot A_4 \\ A_4 &= q_{41} \odot A_1 + q_{43} \odot A_3 + q_{45} \odot A_5 \\ A_5 &= q_{53} \odot A_3 \end{aligned} \tag{6-11}$$

By solving above equations, we get the value of availability A_0 as:

$$A_0 = - \frac{N_2(1)}{D_2'(1)}$$

where

$$\begin{aligned} N_2(1) &= (\nu_3 + \nu_0 P_{30}) (1 - P_{41} + P_{41} P_{13}) + \nu_1 [P_{01} (1 - P_{34} + P_{34} P_{41}) + P_{02} (P_{41} + P_{31} - P_{31} P_{41})], \\ D_2'(1) &= - \{ (\nu_3 + \nu_0 P_{30}) (1 - P_{41} + P_{41} P_{13}) + \nu_1 [P_{01} (1 - P_{34} + P_{34} P_{41}) + P_{02} (P_{41} + P_{31} - P_{31} P_{41})] \\ &+ \nu_2 \{ P_{01} P_{12} (1 - P_{34} + P_{34} P_{41}) + P_{02} [P_{30} (1 - P_{14} P_{41}) + P_{12} (P_{31} + P_{34} P_{41})] \} + (\nu_4 + \nu_5 P_{45}) (1 - P_{13} P_{31} - P_{13} P_{30} P_{01}) \} \end{aligned} \tag{12-13}$$

6. ANALYSIS OF DOWN PERIOD OF SYSTEM

The down period of system i.e in failed state of units the states were divided into two categories accordingly as:

- a) when failed unit follows inspection mechanism.
- b) repair period of failed parts by replacing with new ones.

6.1 Down period of Inspector

Let us denote B_i to be the probability of inspection period in which exact failure is analyzed and communicated to repairman that been corrected by replacing with new one. Using probabilistic argument, the relations obtained are:

$$\begin{aligned}
 B_0 &= q_{01} \odot B_1 + q_{02} \odot B_2 \\
 B_1 &= Z_1 + q_{12} \odot B_2 + q_{13} \odot B_3 + q_{14} \odot B_4 \\
 B_2 &= Z_2 + q_{24} \odot B_4 \\
 B_3 &= q_{30} \odot B_0 + q_{31} \odot B_1 + q_{34} \odot B_4 \\
 B_4 &= Z_4 + q_{41} \odot B_1 + q_{43} \odot B_3 + q_{45} \odot B_5 \\
 B_5 &= q_{53} \odot B_3
 \end{aligned}
 \tag{14-19}$$

solving the above equations, we get the value of busy schedule of inspection B_0 as:

$$B_0 = - \frac{N_3(1)}{D'_2(1)}$$

where

$$\begin{aligned}
 N_3(1) &= \aleph_1 \{P_{01} (1 - P_{34} + P_{34}P_{41}) + P_{02} (P_{41} + P_{31} - P_{31}P_{41})\} + \aleph_2 \{P_{01}P_{12}(1 - P_{34} + P_{34}P_{41}) + \\
 &P_{02}[P_{30}(1 - P_{14}P_{41}) + P_{12}(P_{31} + P_{34}P_{41})]\} + \aleph_4 (1 - P_{13}P_{31} - P_{13}P_{30}P_{01}).
 \end{aligned}
 \tag{20}$$

6.2 Down Period of Repairman

Let us denote B_i' to be the probability of inspection period in which exact failure is analyzed and communicated to repairman that been corrected by replacing with new one. Using probabilistic argument, the relations obtained are:

$$\begin{aligned}
 B'_0 &= q_{01} \odot B'_1 + q_{02} \odot B'_2 \\
 B'_1 &= q_{12} \odot B'_2 + q_{13} \odot B'_3 + q_{14} \odot B'_4 \\
 B'_2 &= q_{24} \odot B'_4 \\
 B'_3 &= Z_3 + q_{30} \odot B'_0 + q_{31} \odot B'_1 + q_{34} \odot B'_4 \\
 B'_4 &= Z_4 + q_{41} \odot B'_1 + q_{43} \odot B'_3 + q_{45} \odot B'_5 \\
 B'_5 &= Z_5 + q_{53} \odot B'_3
 \end{aligned}
 \tag{21-26}$$

solving the above equations, we get the value of busy schedule of inspection B_0' as:

$$B'_0 = - \frac{N_3(1)}{D'_2(1)}$$

where

$$N_4(1) = \aleph_3(1 - P_{41} + P_{41}P_{13}) + (\aleph_4 + \aleph_5P_{45})(1 - P_{13}P_{31} - P_{13}P_{30}P_{01}). \tag{27}$$

7. CONCLUSION

The total profit of system to in steady-state will be calculated by using the relation:

$$P = C_0A_0 \bar{\kappa} C_1B_0 - C_2B'_0 \tag{28}$$

where

C_0 : the cost per unit of operative time by the system.

C_1 & C_2 : the cost per unit of down/failure time expenditure spent for repairing system.

As per the data analysis, the performance of profit function was analyzed through having some fixed parameters as C_0, C_1, C_2, p_1, R as

$$C_0 = 2000, C_1 = 100, C_2 = 500, p_1 = 0.35, 0.4, 0.45 \text{ and } R = 0.05, 0.1, 0.15$$

The behavior of reliability measures of system effectiveness (Table 2 and 3) for above values are calculated as:

Profit w. r. t Failure rate

For $R = 0.05$, the profit function $P > \text{ or } = \text{ or } < 0$ as the failure rate $p_1 < \text{ or } = \text{ or } > 0.16782$.

For $R = 0.1$, the profit function $P > \text{ or } = \text{ or } < 0$ as the failure rate $p_1 < \text{ or } = \text{ or } > 0.314$.

For $R = 0.15$, the profit function $P > \text{ or } = \text{ or } < 0$ as the failure rate $p_1 < \text{ or } = \text{ or } > 0.4407$.

Table 2. Reliability parameters w.r.t Repair rate (R)

Repair	MTSF	A_0	B_0	B'_0	PROFIT (P)
R =0.05	15.70136	0.371035	0.322125	0.843863	287.92537
	10.07065	0.244344	0.400732	0.801465	47.882641
	7.377622	0.19129	0.409198	0.818396	-67.53706
	5.795837	0.157106	0.414197	0.828394	-141.405
	4.752636	0.133272	0.417481	0.834962	-192.6851
	4.011118	0.115715	0.419797	0.839594	-230.3459
	3.455506	0.102249	0.421516	0.843032	-259.169
	3.022468	0.091596	0.422842	0.845684	-281.9346
R =0.1	16.36711	0.559998	0.455588	0.667995	740.44013
	10.32467	0.348253	0.592431	0.592431	341.04839
	7.5	0.278846	0.614103	0.614103	189.23077
	5.862816	0.232332	0.627268	0.627268	88.302571
	4.792323	0.199065	0.636065	0.636065	16.490474
	4.035906	0.174119	0.642341	0.642341	-37.16562
	3.47155	0.154733	0.647037	0.647037	-78.75623
	3.033101	0.13924	0.65068	0.65068	-111.9283
R =0.15	16.8543	0.665974	0.521821	0.542945	1008.2945
	10.52125	0.395519	0.686315	0.457543	493.63497
	7.59887	0.319997	0.716213	0.477475	329.63594
	5.918807	0.26842	0.734502	0.489668	218.55537
	4.826432	0.231075	0.746761	0.49784	138.55435
	4.057706	0.20283	0.755517	0.503678	78.269342
	3.485936	0.180739	0.762071	0.508048	31.246439
	3.042793	0.162998	0.767156	0.511438	-6.439036

Figure 2 reflects the decreasing performance of profit 'P' w.r.t increase in failure p_1 for distinct repairing (R) rate.

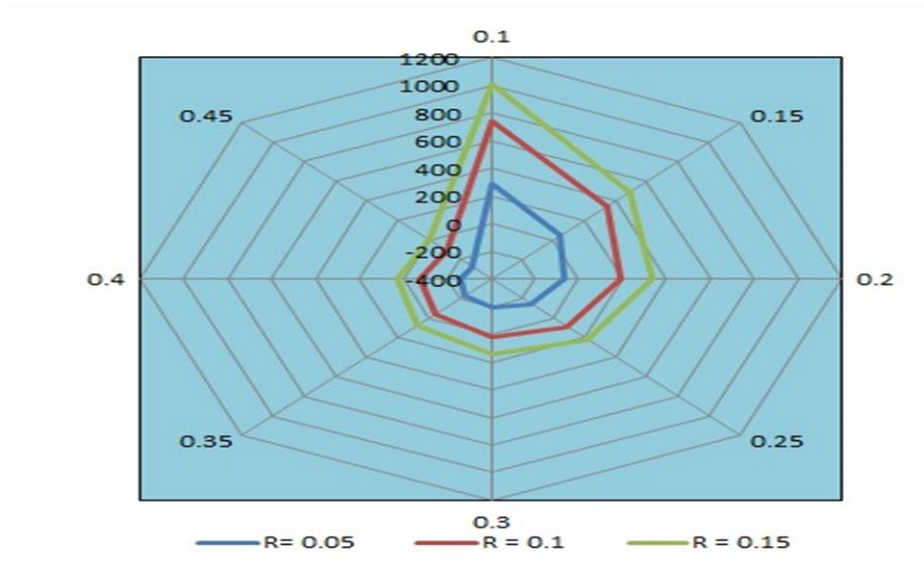


Figure 2. Profit vs Failure Rate

Profit w. r. t Repair rate:

For $p_1 = 0.35$, the profit function $P > \text{ or } = \text{ or } < 0$ as the repair rate $R > \text{ or } = \text{ or } < 0.10332$

For $p_1 = 0.4$, the profit function $P > \text{ or } = \text{ or } < 0$ as the repair rate $R > \text{ or } = \text{ or } < 0.13316$

For $p_1 = 0.45$, the profit function $P > \text{ or } = \text{ or } < 0$ as the repair rate $R > \text{ or } = \text{ or } < 0.15396$.

Table 3. Reliability parameters w.r.t Failure rate (p_1)

p_1	MTSF	A_0	B_0	B'_0	PROFIT (P)
$p_1 = .35$	4.035906	0.23758	0.511986	0.876453	-14.26458
	4.057706	0.20283	0.755517	0.503678	78.269342
	4.077027	0.218351	0.818441	0.40922	150.24712
	4.09427	0.227547	0.856659	0.342663	198.09738
	4.109753	0.233414	0.881589	0.293863	231.73837
	4.123733	0.237381	0.898792	0.256798	256.48351
	4.136417	0.240189	0.911203	0.227801	275.35669
	4.147979	0.242252	0.920484	0.204552	290.17903
$p_1 = .4$	3.47155	0.212889	0.514375	0.890226	-70.77174
	3.485936	0.180739	0.762071	0.508048	31.246439
	3.498908	0.194835	0.825956	0.412978	100.58544
	3.510666	0.203189	0.864643	0.345857	146.98577
	3.521372	0.208513	0.889786	0.296595	179.74987
	3.531161	0.212106	0.907066	0.259162	203.92434
	3.540146	0.214644	0.919482	0.229871	222.40442
	3.548422	0.216504	0.928729	0.206384	236.94358

	3.033101	0.192853	0.516177	0.901219	-116.5212
	3.042793	0.162998	0.767156	0.511438	-6.439036
	3.051666	0.175904	0.831773	0.415886	60.686715
	3.059818	0.183553	0.870802	0.348321	105.86626
$p_1 = .45$	3.067333	0.188424	0.896087	0.298696	137.89162
	3.074285	0.191706	0.913405	0.260973	161.58599
	3.080733	0.194021	0.925805	0.231451	179.73621
	3.086731	0.195714	0.935007	0.207779	194.03852

Figure 3 reflects the increasing performance of profit ' P ' w.r.t repairing (R) for distinct failure p_1 rate.

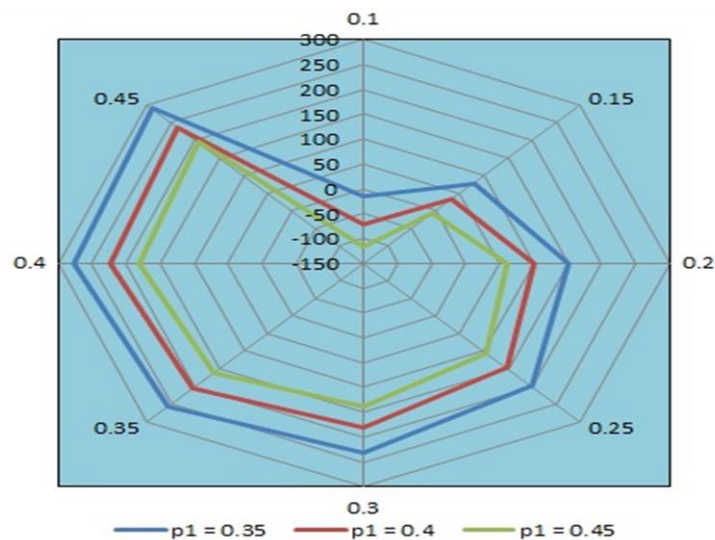


Figure 3. Profit vs Repair Rate

Thus, this paper conclude that the behavior of all reliability parameter can be studied according to different environmental conditions for any other model based on this concept.

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9. REFERENCES

- Adlakha, N., Taneja, G., Shilpi. (2017). Reliability and cost-benefit analysis of a two-unit cold standby system used for communication through satellite with assembling and activation time, *International Journal of Applied Engineering Research*, 12(20): 9697–9702.
- Agarwal, S.C., Sahani, M., Bansal, S. (2010). Reliability characteristic

- of cold-standby redundant system”, *International Journal of Recent Research and Applied Studies*, 3(2): 193–199.
- Bhardwaj, N., Kumar, A., Kumar, S. (2008). Stochastic analysis of a single unit redundant system with two kinds of failure and repairs, *Reflections des. ERA-JMS*, 3(2): 115–134.
- Bhardwaj, N. (2009). Analysis of two-unit redundant system with imperfect switching and connection time, *International Transactions in Mathematical Sciences and Computer*, 2(2): 195–202.
- Bhardwaj, N., Bhatti, J., Parashar, B., Sharma, N. (2015). Stochastic analysis of standby system with different failure, *International Journal of Applied Engineering Research*, 10(35): 27553–27563.
- Bhardwaj, N., Parashar, B. (2017). Stochastic Reliability Analysis of Two Identical Cold Standby Units with Geometric Failure and Repair Rates, *Mathematical Journal of Interdisciplinary Sciences*, 5(2): 131-142.
- Bhatti, J., Chitkara, A., Kakkar, M. (2014). Stochastic analysis of parallel system with two discrete failures, *Model Assisted Statistics and Applications*, 9: 257-265.
- Bhatti, J., Chitkara, A., Kakkar, M. (2016). Stochastic analysis of dis-similar standby system with discrete failure, inspection and replacement policy, *Demonstratio Mathematica*, 49(2): 224–235.
- Bhatti, J., Kakkar, M. (2020). Reliability analysis of cold standby parallel system possessing failure and repair rate under geometric distribution, *Recent Advances in Computer Science and Communications*, 13(5): 1-7.
- Gupta, S., Gupta, S.K. (2013). Stochastic analysis of a reliability model of one-unit system with post inspection, post repair, preventive maintenance and replacement, *International Journal of Mechanical Engineering and Robotics Research*, 2(2): 178–188.
- Haggag, M.Y. (2009). Cost Analysis of a System Involving Common Cause Failures and Preventive Maintenance, *J. Math and Stat.*, 5(4): 305–310.
- Haggag, M.Y. (2009). Cost analysis of two-dissimilar unit cold standby system with three states and preventive maintenance using linear first order differential equations, *J. Math and Stat.*, 5(4): 95–400.
- Kakkar, M. K., Bhatti, J. (2015). Reliability and profit analysis of standby unit system with correlated life time in an industry, *Advance Study in Contemporary Mathematics*, 25(3): 333–340.
- Kakkar, M. K., Chitkara, A. K., Bhatti, J. (2015). Reliability analysis of two-unit parallel repairable industrial system, *Decision Science Letters*, 4: 525–536.
- Kakkar, M.K., Chitkara, A.K., Bhatti, J. (2016). Reliability analysis of two

- dissimilar parallel unit repairable system with failure during preventive maintenance, *Management Science Letters*, 6: 285-296.
- Kakkar, M.K, Bhatti, J, Malhotra, R., Kaur, M. Goyal, D. (2019). Availability Analysis of an Industrial System under the Provision of Replacement of a Unit using Genetic Algorithm, *International Journal of Innovative Technology and Exploring Engineering (IJITEE)*, 9: 1236-1241.
- Malhotra, R., Taneja, G. (2014). Stochastic analysis of a two-unit cold standby system wherein both units may become operative depending upon the demand, *Journal of Quality and Reliability Engineering*, 1–13.
- Rizwan, S. M., Khurana, V., Taneja, G. (2010). Reliability Analysis of a Hot Standby Industrial System, *International Journal of Modelling and Simulation*, 30(3): 315–322.
- Said, K.M.EL., Salah, M., Sherbeny, EL. (2005). Profit analysis of a two-unit cold standby system with preventive maintenance and random change in units. *Journal of Mathematics and Statistics*, 1(1): 71–77.
- Singh, D., Taneja, G. (2014). Reliability and economic analysis of a power generating system comprising one gas and one steam turbine with random inspection, *Journal of Mathematics and Statistics*, 10(4): 436–442.
- Taj, S.Z., Rizwan, S.M., Alkali, B.M., Harrison, D.K., Taneja, G. (2017). Probabilistic modeling and analysis of a cable plant subsystem with priority to repair over preventive maintenance, *I-Managers Journal on Mathematics*, 6(3): 12–21.