Control Charts with Robust Probability Limits

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ABSTRACT Two of the main problems in constructing a control chart for detecting shifts in process variation are to estimate the process variation based on preliminary samples taken from the process and to evaluate its control limits. The unknown process variation is generally estimated from either the sample standard deviations or ranges of the preliminary samples. These classical estimates of process variation are highly sensitive to the presence of contaminated data in the preliminary samples and subsequently reduce the power of control charts in detecting assignable causes. The 3-sigma control limits of the Shewhart control charts are evaluated based on the assumption that the sample statistic being plotted is Gaussian distributed. However, the sampling distributions of the sample standard deviation and range are skewed even if the samples are taken from a Gaussian population. The aims of this paper are (i) to discuss robust estimates of scale parameter from preliminary samples taken from the process under study, and (ii) to construct control charts with probability limits evaluated using robust estimators that are resistance to contaminated preliminary samples.

ABSTRAK Dua masalah utama dalam pembinaan carta kawalan bagi mengesan perubahan dalam serakan proses adalah menganggar serakan proses berasaskan sampel awalan yang diambil dari proses, dan menentukan had kawalannya. Lazimnya, serakan proses yang tidak diketahui diukur dengan sisihan piawai atau julat sampel awalan. Penganggar klasik bagi serakan proses adalah sangat sensitif kepada kewujudan data tercemar dalam sampel awalan, dan seterusnya mengurangkan kuasa carta kawalan dalam mengesan penyebab umpukan. Carta kawalan 3-sigma untuk carta kawalan Shewhart ditentukan berdasarkan andaian bahawa sampel statistik yang dikaji adalah bertaburan Gaussian. Bagaimanapun, taburan pensampelan bagi sisihan piawai sampel dan julat sampel adalah pencong walaupun sampel dipilih dari populasi Gaussian. Tujuan kertas ini adalah (i) membincang penganggar kukuh bagi parameter skel dari sampel awalan yang diambil dari proses dikaji, dan (ii) membina carta kawalan dengan had kebarangkalian yang dinilai dengan menggunakan penganggar kukuh yang mempunyai rintangan terhadap sampel awalan tercemar.

(Robust estimator, robust control chart, asymmetric control limits)

INTRODUCTION

Control chart is a basic tool in statistical process control for detecting shifts in process mean and variation. The standard Shewhart control charts are constructed under the normality assumption. Let w be a sample statistic that measures a quality characteristic of interest in the process under study, and let that the mean of w be μ_w and the standard deviation of w be σ_w. Then the center line, the upper and lower control limits of the standard Shewhart control chart for the sample statistic w are given as

$$UCL_w = \mu_w + L\sigma_w$$
$$CL_w = \mu_w$$
$$LCL_w = \mu_w - L\sigma_w$$

where the factor L is usually taken to be 3.0 irrespective of the distribution of w. However, the sampling distributions of the sample range and sample standard deviation are fairly skewed, even for samples taken from a Gaussian distribution, thus the control limits of the R-chart and s-chart set at plus and minus L standard deviations of its mean value are inappropriate. It has been pointed out that for highly skewed process, samples of size four or five are not sufficient to satisfy the normality assumption of the sample mean [1]. The study in [2] also concluded that the false alarm rate of out-of-control signals would be greatly increased if the normality assumption of the sample mean is violated. To avoid this drawback, one should construct the Shewhart charts using the asymmetric probability limits instead of the 3-sigma control limits. The construction of s and R charts with asymmetric probability limits, evaluated based on additional knowledge on the distribution of the process under study, are available and can be found in [3, 4, 5].

Another problem of the Shewhart control charts that have attracted a lot of attention lately is on the estimation of the unknown process mean μ and standard deviation σ. In most applications, the location and scale parameters are estimated from preliminary subgroups taken from the process under study when it is operating in the state of statistical control with only chance causes of variation present. The most widely used estimator of the location parameter is the average of the m sample means evaluated from the m preliminary subgroups. The scale parameter is usually estimated based on the average of the m sample ranges, the average of the m sample standard deviations or the square of the average of the m sample variances. However, these classical unbiased estimates of location and scale parameters are greatly affected by the presence of contaminated and outlying data in the preliminary samples. An inflated estimate of σ would result in wider control limits and subsequently reduces the power of control chart in detecting out-of-control signals. A deflated estimate of σ would result in shorter control limits that lead to higher false alarm rate of detecting assignable causes. To avoid this drawback, an alternative is to construct a control chart based on a sample statistic (e.g. sample mean, standard deviation or range) that is sensitive to out-of-control signal, however, with its control limits evaluated using robust estimators that are resistant to contaminated sample data. A good robust estimator is efficient and resistant. High efficiency implies that the sampling distribution of the estimator has small variance even when we are sampling from a non-normal distribution. An estimator is resistant if small changes in some of the sample data or large changes in a few of the data values have a small effect on its estimate value.

The aims of this paper are (i) to discuss robust estimates of scale parameter from preliminary samples taken from the process under study, and (ii) to construct control charts with probability limits evaluated using robust estimators that are resistance to contaminated preliminary samples. The construction of s, R and $\bar{x}$ charts with probability limits evaluated from samples taken from the Gaussian and selected non-Gaussian populations are discussed in Section 2. Some well develop robust scale estimators for the Gaussian and exponential distributions, as well as a resistant biweight estimator that can be used to estimate the scale parameter of a family of location-scale populations are discussed in Section 3. Our simulation study reveals that control charts with resistant probability limits outperform that constructed with the classical non-resistant scale estimator when the preliminary samples are contaminated. To avoid lengthy discussions, we only report the performance of the s-chart with robust probability limits in Section 4.

SHEWHART CONTROL CHARTS WITH PROBABILITY LIMITS

For a process with known scale parameter σ, the center line and $(1-\alpha)100\%$ probability limits of the s-chart for samples of size n are defined as

$$UCL_s = B_{1-\alpha/2;n}\sigma$$
$$CL_s = c_n\sigma$$
$$LCL_s = B_{\alpha/2;n}\sigma$$

and that of the R-chart are given as

$$UCL_R = D_{1-\alpha/2;n}\sigma$$
$$CL_R = d_n\sigma$$
$$LCL_R = D_{\alpha/2;n}\sigma$$

where $B_{p,n}$ is the pth percentage point of $V = s/\sigma$ and $c_n = E(V)$; $D_{p,n}$ is the pth percentage point of the relative sample range $U = R/\sigma$ and $d_n = E(U)$.

For samples taken from the exponential, Laplace or logistic distributions, the values of $B_{.00135;n}$, $B_{.99865;n}$ and c_n are given in [4], whereas the values of $D_{.00135;n}$, $D_{.99865;n}$ and d_n are given in [5] for selected sample size n. For the case when samples are taken from the Gaussian distribution, the selected percentage points of R/σ can be obtained from the tables given in [6], whereas the $(\alpha/2)$th and $(1-\alpha/2)$th percentage points of s/σ are

$$B_{\alpha/2;n} = \sqrt{\chi^2_{\alpha/2;n-1}/(n-1)}$$

and

$$B_{1-\alpha/2;n} = \sqrt{\chi^2_{1-\alpha/2;n-1}/(n-1)}$$

respectively. These values of B's and D's are larger than the corresponding factor values of the classical s and R charts constructed with 3-sigma control limits under the normality assumption. A salient feature of these two control charts is that its lower probability limits LCL_s and LCL_R are positive even for sample of size two to five.

For a process with known mean μ and variance σ^2, the center line and probability limits of the $\bar{x}$-chart for samples of size n are given as

$$UCL_{\bar{x}} = \mu + L_{1-\alpha/2;n}(\sigma/\sqrt{n})$$
$$CL_{\bar{x}} = \mu$$
$$LCL_{\bar{x}} = \mu - L_{\alpha/2;n}(\sigma/\sqrt{n})$$

where $L_{1-\alpha/2;n}$ and $L_{\alpha/2;n}$ are control factors that depend on the sampling distribution of the sample mean and a specified false alarm rate of α. For samples taken from the Gaussian $N(\mu,\sigma^2)$ distribution, the values of $L_{1-\alpha/2;n}$ and $L_{\alpha/2;n}$ corresponding to $\alpha = 0.0027$ are both taken to be 3.0 irrespective of the value of n. For samples taken from the $Laplace(\theta,\beta)$ distribution with mean $\mu = \theta$ and $\sigma = \sqrt{2}\beta$, the control factors $L_{p;n}$ are taken to be the pth percentage point of the sampling distribution of its sample mean which is distributed as the difference of two IID gamma random variables with shape parameter n and scale parameter β/n. For sample taken from the $Gamma(\beta,\nu)$ distribution with mean $\mu = \nu\beta$ and standard deviation $\sigma = \beta\sqrt{\nu}$, the control factors are given in [7] as $L_{1-\alpha/2;n} = (G_{1-\alpha/2}/\sqrt{n\nu}) - \sqrt{n\nu}$ and $L_{\alpha/2;n} = \sqrt{n\nu} - (G_{\alpha/2}/\sqrt{n\nu})$ where G_p is the pth percentage point of a gamma distribution with shape parameter $n\nu$ and scale parameter equal to 1. The values of $L_{1-\alpha/2;n}$ and $L_{\alpha/2;n}$ for sample of size n taken from the Laplace and exponential distributions are given in Table 1 for $n = 2(1)20$ and 50. Examination of Table 1 reveals that for samples of size as large as 50 taken from the Laplace or the exponential distributions, the values of $L_{1-\alpha/2;n}$ and $L_{\alpha/2;n}$ are not close to its corresponding value of 3.0 as expected under the normality assumption.

Table 1. Factors for constructing the 99.73% probability limits of the $\overline{x}$ chart for samples of size n taken from the Laplace distribution or exponential distribution

n	LAPLACE DISTRIBUTION $L_{.00135;n} = L_{.99865;n}$	EXPONENTIAL DISTRIBUTION $L_{.00135;n}$	EXPONENTIAL DISTRIBUTION $L_{.99865;n}$	n	LAPLACE DISTRIBUTION $L_{.00135;n} = L_{.99865;n}$	EXPONENTIAL DISTRIBUTION $L_{.00135;n}$	EXPONENTIAL DISTRIBUTION $L_{.99865;n}$
2	3.7347	1.1494	5.1067	12	3.1685	2.1584	3.8720
3	3.5422	1.4217	4.7317	13	3.1567	2.1902	3.8378
4	3.4322	1.6034	4.5042	14	3.1465	2.2185	3.8074
5	3.3603	1.7348	4.3476	15	3.1375	2.2440	3.7800
6	3.3094	1.8325	4.2313	16	3.1295	2.2672	3.7552
7	3.2713	1.9150	4.1407	17	3.1224	2.2883	3.7326
8	3.2417	1.9804	4.0674	18	3.1161	2.3078	3.7119
9	3.2179	2.0353	4.0066	19	3.1104	2.3257	3.6929
10	3.1985	2.0822	3.9551	20	3.1052	2.3423	3.6754
11	3.1823	2.1228	3.9107	50	3.0437	2.5802	3.4266

SOME ROBUST AND RESISTANT SCALE ESTIMATORS

It is well established that a robust estimator is able to perform well for its intended purpose even if the underlying assumptions on which it is based are violated [8]. A robust estimator is resistance if it is affected to only a limited extent by the presence of contaminated data or outlying observations. Ideally outlying observations should first be identified using formal hypothesis testing procedures [9] or graphical procedures [10]. However, the outlier detection procedures are in general incapable of detecting small changes in contaminated sample. Therefore in the event that the process standard deviation σ is not available when constructing a control chart, one should estimate the unknown σ by using resistant scale estimators that are able to accommodate and reduce the influence of contaminated data and outlying observations.

There are many robust estimators of location and scale parameters. Some of the widely used robust location estimators are the median, trimmed mean and M-estimator. An efficient median estimator defined as the weighted sum of ordered sample data with symmetric weights has also been discussed [11]. In this paper, we shall focus on the effect of the resistant scale parameter on the performance of the control charts. Three of the commonly used robust scale estimators are the median absolute deviation about the median (MAD) [12], the S_n and Q estimators [13]. For sample of size n, these estimators are defined as

$$MAD = b_n Med_i(|x_i - Med_j x_j|, i, j = 1,2,\ldots,n)$$

$$S_n = s_n Med_i\{Med_j(|x_i - x_j|), i \neq j; i, j = 1,2,\ldots,n\}$$

$$Q = q_n(|x_i - x_j|; i < j; i, j = 1,2,\ldots,n)_k$$

where $(\ )_k$ is the k-th order statistic of the interpoint distances $|x_i - x_j|$ with $k = C_2^n/4$. The constants b_n, s_n and q_n are correction factors chosen to ensure that the respective estimator is unbiased to the scale parameter σ. For large Gaussian sample, these correction factors are taken to be $b_n = 1.4826$, $s_n = 1.1926$ and $q_n = 2.2219$ [13]. For large exponential sample, they are taken to be $s_n = 1.6982$ and $q_n = 3.476$ [14]. For finite Gaussian sample of size n, approximate values of these correction factors are also available in [14]. These three estimators have the highest possible breakdown point of 50%, i.e. the estimate of the scale parameter σ remains bounded when fewer than 50% of the data points are replaced by arbitrary values. In contrast, the commonly used sample range and sample standard deviation have breakdown point of 0%.

In constructing control charts, a common approach is to estimate the process variability based on preliminary samples taken during a trial period when the process is operating under the state of statistical control. A robust scale estimator that estimates the process standard deviation based on m preliminary Gaussian samples was proposed in [15] using the biweight A-estimator [16]. Let $Med_i(x_{k,i})$ and IQR_k be the median and interquartile range of the kth sample with n data $x_{k,1}, x_{k,2}, \ldots, x_{k,n}$, the scale estimator in [15] is defined as

$$S_c^* = \frac{N}{(N-1)^{1/2}} \frac{\left[\sum_k \sum_{i:|u_{k,i}|<1} y_{k,i}^2 (1-u_{k,i}^2)^4\right]^{1/2}}{\left|\sum_k \sum_{i:|u_{k,i}|<1} (1-u_{k,i}^2)(1-5u_{k,i}^2)\right|} \quad (1)$$

where $N = mn$ when n is even, $N = m(n-1)$ when n is odd, $u_{k,i} = h_k y_{k,i} / cMAD_N$ in which c is a constant that lies in the range 6 to 12, MAD_N is the MAD of the N median-centered subsample values $y_{k,i} = x_{k,i} - Med_i(x_{k,i})$, and

$$h_k = \begin{cases} 1, & E_k \le 4.5 \\ E_k - 4.5, & 4.5 < E_k \le 7.5 \\ c, & E_k > 7.5 \end{cases} \quad (2)$$

with $E_k = IQR_k / MAD_N$. Note that for Gaussian samples with large sample size, we have $E(MAD) \approx \frac{2}{3}\sigma$, thus an estimator S_c^* with $c = 9$ implies that observations with magnitude more than $\frac{2}{3}(9) = 6$ standard deviations away from the median will not be included in the sum of equation (1). The process standard deviation σ is then estimated as

$$S_c = S_c^* / d_{n,m,c} \quad (3)$$

where $d_{n,m,c}$ is a correction factor chosen to ensure that $E(S_c) = \sigma$. The cutoff values of E_k in equation (2) were determined under the normality assumption by the authors via simulation studies. The values of $d_{n,m,c}$ are given in Table 2 for $c = 7$ and selected values of n and m.

A drawback of the estimator S_c is that in computing $u_{k,i}$, the constant c and the cutoff values of E_k cannot be determined analytically under the hypothesized distribution. To overcome this drawback, we propose an alternative estimator, denoted S_{box}^*, defined as in (1) but with its relative deviation $u_{k,i}$ replaced by

$$u_{k,i} = \begin{cases} \dfrac{Med_i(x_{k,i}) - x_{k,i}}{Med_i(x_{k,i}) - LF_k}, & x_{k,i} \le Med_i(x_{k,i}) \\ \dfrac{x_{k,i} - Med_i(x_{k,i})}{UF_k - Med_i(x_{k,i})}, & x_{k,i} > Med_i(x_{k,i}) \end{cases}$$

where $LF_k = x_{l:n} - k_l(x_{u:n} - x_{l:n})$ and $UF_k = x_{u:n} + k_u(x_{u:n} - x_{l:n})$ in which $x_{l:n}$ and $x_{u:n}$ are the lower-fourth and upper-fourth of the kth sample of size n. The values of k_l, k_u are determined based on the requirement that for an outlier-free random sample taken from the hypothesized distribution, the probability that one or more observations in the sample will be wrongly classified as outliers is equal to a prescribed small value α_0 [10]. The values of $k_l = k_u \ (= k)$ for sample taken from the standard Gaussian distribution are given in Table 2 for $\alpha_0 = 0.01$, 0.05 and selected sample size n. The process standard deviation σ is then estimated as

$$S_{box} = S_{box}^* / s_{n,m,\alpha_0} \quad (4)$$

where s_{n,m,α_0} is a correction factor chosen to ensure that $E(S_{box}) = \sigma$. The values of s_{n,m,α_0} are given in Table 2 for $\alpha_0 = 0.01$ and 0.05, and selected values of n and m.

A salient feature of the estimator S_{box} is that in computing $u_{k,i}$, the required values of k_l and k_u can be evaluated explicitly not only for the Gaussian distribution but also for the family of location-scale distributions. For example, the proposed estimator S_{box} can be used to estimate the scale parameter of the asymmetric Laplace, logistic distributions and the asymmetric exponential, extreme-value distributions. For Gaussian and exponential samples of size 9 to 500, the values of k_l and k_u is given in [10].

Table 2. The correction factors $d_{n,m,c}$ of the estimator S_c with c=7, the values of $k_l = k_u$ (=k) and correction factors s_{n,m,α_0} of the estimator S_{box} for m preliminary samples of size n taken from the $N(0,1)$ distribution. Note that α_0 is the error rate that one or more observations in an outlier-free sample would be wrongly classified as outlier(s). The values of $d_{n,m,c}$ and s_{n,m,α_0} are obtained from 100,000 replications of m preliminary samples generated from the $N(0,1)$ distribution

SAMPLE SIZE, n	Mod (n,4)	$\alpha_0 = 0.05$				$\alpha_0 = 0.01$				c = 7.0		
			s_{n,m,α_0}				s_{n,m,α_0}			$d_{n,m,c}$		
		k	m=50	m=100	m=200	k	m=50	m=100	m=200	m=50	m=100	m=200
9	1	3.383	0.94740	0.94751	0.94749	5.783	0.92180	0.92174	0.92162	0.90623	0.90595	0.90578
13		2.870	0.98677	0.98674	0.98676	4.349	0.95519	0.95504	0.95008	0.94450	0.94417	0.94405
17		2.650	1.00799	1.00797	1.00806	3.772	0.97422	0.97410	0.97412	0.96423	0.96397	0.96393
21		2.531	1.02113	1.02120	1.02122	3.464	0.98668	0.98666	0.98664	0.97622	0.97608	0.97602
25		2.458	1.02997	1.03002	1.03002	3.275	0.99549	0.99547	0.99546	0.98434	0.98419	0.98412
10	2	2.563	1.01141	1.01156	1.01153	4.104	0.97861	0.97858	0.97847	0.96569	0.96550	0.96530
14		2.424	1.03098	1.03100	1.03106	3.560	0.99557	0.99545	0.99546	0.98389	0.98365	0.98355
18		2.351	1.04172	1.04173	1.04181	3.286	1.00556	1.00550	1.00553	0.99399	0.99377	0.99375
22		2.309	1.04831	1.04833	1.04839	3.123	1.01230	1.01225	1.01226	1.00029	1.00014	1.00009
26		2.282	1.05264	1.05270	1.05270	3.015	1.01706	1.01707	1.01704	1.00450	1.00439	1.00431
11	3	2.091	0.98268	0.98280	0.98275	3.209	0.94656	0.94653	0.94644	0.93139	0.93119	0.93104
15		2.114	1.00702	1.00695	1.00699	3.036	0.97026	0.97011	0.97010	0.95625	0.95596	0.95587
19		2.124	1.03970	1.02112	1.02118	2.926	0.98457	0.98450	0.98451	0.97097	0.97078	0.97073
23		2.130	1.03029	1.03040	1.03043	2.852	0.99429	0.99430	0.99430	0.98063	0.98052	0.98046
27		2.135	1.03674	1.03681	1.03680	2.800	1.00136	1.00137	1.00133	0.98743	0.98735	0.98724
12	0	2.266	1.08395	1.08413	1.08428	3.355	1.01936	1.01941	1.01945	0.97534	0.97507	0.97494
16		2.254	1.08362	1.08379	1.08389	3.169	1.02552	1.02557	1.02558	0.98945	0.98925	0.98915
20		2.239	1.08293	1.08303	1.08314	3.040	1.02958	1.02957	1.02961	0.99748	0.99727	0.99723
24		2.231	1.08168	1.08181	1.08185	2.956	1.03207	1.03211	1.03209	1.00259	1.00245	1.00238
28		2.226	1.08040	1.08054	1.08055	2.895	1.03381	1.03388	1.03386	1.00610	1.00602	1.00594

PERFORMANCES OF THE ROBUST SCALE ESTIMATORS

We shall now examine the performance of the robust scale estimators on preliminary samples taken from the following types of contaminated Gaussian distributions.

Type 1: $CN(p,a)$ distribution
Each of the sample data has $100(1-p)\%$ probability of being drawn from the $N(0,1)$ distribution and $100p\%$ probability of being drawn from the $N(0,a^2)$ distribution with $a \geq 1$.

Type 2: $CSlash(p)$ distribution
Each of the sample data has $100(1-p)\%$ probability of being drawn from a $N(0,1)$ distribution and $100p\%$ probability of being drawn from the long tailed Slash distribution, defined as the $N(0,1)$ random variable divided by an independent uniform random variable on the interval $(0,1)$.

Type 3: $C\chi^2(p,c)$ distribution
Each of the sample data is drawn from the $N(0,1)$ distribution and has a $100p\%$ probability of adding to it a value cV, where V is drawn from the chi-square distribution with one degree of freedom and c is a positive constant value. Note that $c = 0$ corresponds to the uncontaminated samples.

The $CN(p,a)$ distribution has tails heavier than that of the standard Gaussian distribution, whereas the $CSlash(p)$ distribution has heavy tails similarly to that of the Cauchy distribution. The $C\chi^2(p,c)$ is a standard Gaussian distribution contaminated by a chi-square distribution with a long right tail. Tables 3 and 4 give the average values of $\bar{s}/c_4$, Q, S_c, S_{box}, S_n and MAD evaluated based on 100,000 replications of m samples of size n taken from the uncontaminated $N(0,1)$ distribution, and the abovementioned contaminated Gaussian distributions. The values in bold and italic are the mean absolute deviation between the estimates of the scale parameter σ and its hypothesized value $\sigma = 1$.

Examination of Table 3 reveals that, based on $m = 200$ preliminary samples, the non-robust estimator $\bar{s}/c_4$ of σ used in constructing the control charts is more sensitive to heavy-tailed distributions than are the robust estimators. For example, for samples of size $n = 20$, the average value of $\bar{s}/c_4$ is 1.3340 for the $C\chi^2(p,c)$ distribution and 5.6148 for the $CSlash(p)$ distribution when $p = 0.05$ and $c = 3.0$, whereas the respective average values of S_{box} are 1.0231 and 1.0221. Table 3 also reveals that the robust estimators, in particular the S_c and S_{box} estimators, have the smallest mean absolute deviation values (in bold, italic and underlined) amongst the estimators considered for samples of size $n = 10$ and $n = 20$ taken from the contaminated $CN(p,a)$, $C\chi^2(p,c)$ and $CSlash(p)$ distributions with $p = 0.02$, 0.05 and 0.10, and $a = c = 3$. Table 4 shows that S_{box} remains the most efficient and resistant scale estimator when the number of samples (of size $n = 20$) used in estimating the process standard deviation reduce to $m = 50$. These two tables also indicate that, as expected, $\bar{s}/c_4$ has the smallest mean absolute deviation when the preliminary samples are uncontaminated Gaussian samples.

Table 3. Comparison of scale estimators for samples taken from uncontaminated or contaminated $N(0,1)$ distributions. The entries are the average values obtained from 100,000 replications of m=200 preliminary samples, each of size n, generated from each of the distributions. The value in bold and italic are the mean absolute deviation between the estimates of the scale parameter and its expected value σ = 1. The underlined values are the smallest mean absolute deviation amongst the estimators considered

p	SCALE ESTIMATOR	SAMPLE SIZE n=10 DISTRIBUTION, a=3.0				SAMPLE SIZE n=20 DISTRIBUTION, a=3.0			
		$N(0,1)$	$CN(p,a)$	$C\chi^2(p,a)$	$CSlash(p)$	$N(0,1)$	$CN(p,a)$	$C\chi^2(p,a)$	$CSlash(p)$
0.02	sbar/c_4	1.00004	1.05996	1.11539	1.88546	1.00014	1.06571	1.13833	2.07607
		0.01348	***0.05999***	***0.11539***	***0.88546***	***0.00925***	***0.06571***	***0.13833***	***1.07606***
	Q	1.00765	1.03503	1.03149	1.02831	1.00052	1.02547	1.02202	1.01924
		0.01804	***0.03617***	***0.03312***	***0.03040***	***0.01117***	***0.02594***	***0.02287***	***0.02051***
	S_c	1.00004	1.01841	1.01448	1.01566	1.00010	1.01727	1.01121	1.01069
		0.01502	***0.02237***	***0.02077***	***0.02220***	***0.01005***	***0.01842***	***0.01400***	***0.01367***
	S_{box}	1.00006	1.02096	1.01384	1.01255	1.00012	1.01368	1.00872	1.00857
		0.01518	***0.02416***	***0.01968***	***0.01880***	***0.01140***	***0.01652***	***0.01370***	***0.01362***
	S_n	0.99303	1.01507	1.01169	1.00955	0.99806	1.01858	1.01531	1.01332
		0.01956	***0.02303***	***0.02165***	***0.02070***	***0.01259***	***0.02057***	***0.01829***	***0.01703***
	MAD	1.00006	1.01866	1.01543	1.01384	1.00013	1.01682	1.01395	1.01252
		0.02082	***0.02630***	***0.02473***	***0.02393***	***0.01475***	***0.02062***	***0.01895***	***0.01818***
0.05	sbar/c_4	1.00004	1.14734	1.28190	3.08679	1.00014	1.16014	1.33401	5.61484
		0.01348	***0.14734***	***0.28190***	***2.08681***	***0.00925***	***0.16013***	***0.33401***	***4.61482***
	Q	1.00765	1.07788	1.06875	1.06077	1.00052	1.06456	1.05532	1.04847
		0.01804	***0.07789***	***0.06877***	***0.06085***	***0.01117***	***0.06455***	***0.05532***	***0.04847***
	S_c	1.00004	1.04822	1.03846	1.03919	1.00010	1.04516	1.02924	1.02751
		0.01502	***0.04843***	***0.03950***	***0.04054***	***0.01005***	***0.04516***	***0.02939***	***0.02772***
	S_{box}	1.00006	1.05515	1.03858	1.03259	1.00012	1.03593	1.02306	1.02215
		0.01518	***0.05523***	***0.03928***	***0.03365***	***0.01140***	***0.03603***	***0.02390***	***0.02310***
	S_n	0.99303	1.04966	1.04107	1.03529	0.99806	1.05086	1.04216	1.03719
		0.01956	***0.05015***	***0.04220***	***0.03711***	***0.01259***	***0.05087***	***0.04223***	***0.03735***
	MAD	1.00006	1.04813	1.04043	1.03551	1.00013	1.04318	1.03576	1.03191
		0.02082	***0.04904***	***0.04212***	***0.03795***	***0.01475***	***0.04334***	***0.03623***	***0.03268***
0.10	sbar/c_4	1.00004	1.28654	1.54269	4.98220	1.00014	1.30772	1.63318	6.65260
		0.01348	***0.28654***	***0.54270***	***3.98218***	***0.00925***	***0.30771***	***0.63318***	***5.65257***
	Q	1.00765	1.15357	1.13422	1.11765	1.00052	1.13360	1.11364	1.09969
		0.01804	***0.15356***	***0.13422***	***0.11765***	***0.01117***	***0.13360***	***0.11363***	***0.09969***
	S_c	1.00004	1.10417	1.08377	1.08268	1.00010	1.09745	1.06319	1.05788
		0.01502	***0.10417***	***0.08379***	***0.08271***	***0.01005***	***0.09745***	***0.06320***	***0.05787***
	S_{box}	1.00006	1.12024	1.09418	1.07018	1.00012	1.07814	1.05111	1.04671
		0.01518	***0.12026***	***0.09418***	***0.07020***	***0.01140***	***0.07814***	***0.05112***	***0.04672***
	S_n	0.99303	1.11126	1.09276	1.08045	0.99806	1.10837	1.08959	1.07906
		0.01956	***0.11126***	***0.09277***	***0.08047***	***0.01259***	***0.10838***	***0.08959***	***0.07907***
	MAD	1.00006	1.10134	1.08619	1.07369	1.00013	1.09030	1.07537	1.06596
		0.02082	***0.10134***	***0.08622***	***0.07377***	***0.01475***	***0.09030***	***0.07538***	***0.06596***

Table 4. Comparison of scale estimators for samples taken from uncontaminated or contaminated $N(0,1)$ distributions. The entries are the average values obtained from 100,000 replications of m preliminary samples, each of size n=20, generated from each of the distributions. The value in bold and italic are the mean absolute deviation between the estimates of the scale parameter and its expected value $\sigma = 1$. The underlined values are the smallest mean absolute deviation amongst the estimators considered

M	SCALE ESTIMATOR	SAMPLE SIZE n=20 DISTRIBUTION, p=0.05, a=3.0				SAMPLE SIZE n=20 DISTRIBUTION, p=0.10, a=3.0			
		$N(0,1)$	$CN(p,a)$	$C\chi^2(p,a)$	$CSlash(p)$	$N(0,1)$	$CN(p,a)$	$C\chi^2(p,a)$	$CSlash(p)$
50	sbar/c_4	1.00017	1.16009	1.33454	3.89222	1.00017	1.30756	1.63409	7.74112
		0.01845	***0.16009***	***0.33455***	***2.89223***	***0.01845***	***0.30755***	***0.63409***	***6.74117***
	Q	1.00056	1.06451	1.05544	1.04824	1.00056	1.13353	1.11381	1.09947
		0.02236	***0.06495***	***0.05635***	***0.04972***	***0.02236***	***0.13353***	***0.11382***	***0.09948***
	S_c	1.00012	1.04515	1.02938	1.02735	1.00012	1.09751	1.06343	1.05764
		0.02009	***0.04646***	***0.03320***	***0.03170***	***0.02009***	***0.09753***	***0.06376***	***0.05812***
	S_{box}	1.00017	1.03585	1.02320	1.02195	1.00017	1.07801	1.05129	1.04661
		0.02282	***0.03979***	***0.03087***	***0.03012***	***0.02282***	***0.07828***	***0.05281***	***0.04856***
	S_n	0.99810	1.05077	1.04236	1.03696	0.99810	1.10828	1.08983	1.07892
		0.02512	***0.05278***	***0.04574***	***0.04150***	***0.02512***	***0.10831***	***0.08998***	***0.07923***
	MAD	1.00018	1.04320	1.03600	1.03179	1.00018	1.09030	1.07571	1.06576
		0.02952	***0.04838***	***0.04324***	***0.04051***	***0.02952***	***0.09067***	***0.07661***	***0.06733***
100	sbar/c_4	1.00011	1.16021	1.33435	3.25007	1.00011	1.30778	1.63355	6.77074
		0.01305	***0.16020***	***0.33435***	***2.25008***	***0.01305***	***0.30778***	***0.63355***	***5.77073***
	Q	1.00051	1.06470	1.05530	1.04853	1.00051	1.13371	1.11355	1.09966
		0.01574	***0.06473***	***0.05538***	***0.04870***	***0.01574***	***0.13370***	***0.11355***	***0.09966***
	S_c	1.00005	1.04531	1.02924	1.02763	1.00005	1.09758	1.06315	1.05785
		0.01417	***0.04547***	***0.03031***	***0.02887***	***0.01417***	***0.09758***	***0.06315***	***0.05788***
	S_{box}	1.00009	1.03605	1.02310	1.02215	1.00009	1.07818	1.05100	1.04673
		0.01608	***0.03701***	***0.02611***	***0.02543***	***0.01608***	***0.07820***	***0.05120***	***0.04701***
	S_n	0.99805	1.05100	1.04215	1.03719	0.99805	1.10849	1.08948	1.07904
		0.01777	***0.05129***	***0.04289***	***0.03836***	***0.01777***	***0.10849***	***0.08948***	***0.07905***
	MAD	0.99948	1.04274	1.03513	1.03130	0.99948	1.08980	1.07459	1.06528
		0.02089	***0.04412***	***0.03763***	***0.03460***	***0.02089***	***0.08981***	***0.07465***	***0.06545***
200	sbar/c_4	1.00014	1.16014	1.33401	5.61484	1.00014	1.30772	1.63318	6.65260
		0.00925	***0.16013***	***0.33401***	***4.61482***	***0.00925***	***0.30771***	***0.63318***	***5.65257***
	Q	1.00052	1.06456	1.05532	1.04847	1.00052	1.13360	1.11364	1.09969
		0.01117	***0.06455***	***0.05532***	***0.04847***	***0.01117***	***0.13360***	***0.11363***	***0.09969***
	S_c	1.00010	1.04516	1.02924	1.02751	1.00010	1.09745	1.06319	1.05788
		0.01005	***0.04516***	***0.02939***	***0.02772***	***0.01005***	***0.09745***	***0.06320***	***0.05787***
	S_{box}	1.00012	1.03593	1.02306	1.02215	1.00012	1.07814	1.05111	1.04671
		0.01140	***0.03603***	***0.02390***	***0.02310***	***0.01140***	***0.07814***	***0.05112***	***0.04672***
	S_n	0.99806	1.05086	1.04216	1.03719	0.99806	1.10837	1.08959	1.07906
		0.01259	***0.05087***	***0.04223***	***0.03735***	***0.01259***	***0.10838***	***0.08959***	***0.07907***
	MAD	1.00013	1.04318	1.03576	1.03191	1.00013	1.09030	1.07537	1.06596
		0.01475	***0.04334***	***0.03623***	***0.03268***	***0.01475***	***0.09030***	***0.07538***	***0.06596***

PERFORMANCE OF CONTROL CHARTS WITH ROBUST PROBABILITY LIMITS

The average run length (ARL) is commonly used as a summary measure in evaluating the performance of control charts. In practice, we require a large in-control ARL value that corresponds to a small false alarm rate, and a small out-of-control ARL value to enable rapid detection of undesirable increase in process variability. Let E denotes the event that a sample statistic w falls either above its estimated upper probability limit $\hat{UCL}_w$ or below its estimated lower probability limit $\hat{LCL}_w$. Let RL_k denotes the run length between occurrences of the events E when the process variation shifted from σ to $k\sigma$. Then, given $\hat{\sigma}$, the conditional distribution of RL_k follows a geometric distribution with parameter $P_k(E \mid \hat{\sigma})$ and the ARL required to detect a shift of σ to $k\sigma$ in the process variation is given by

$$ARL(k) = E(RL_k) = \int_0^\infty \frac{1}{P_k(E \mid \hat{\sigma})} f_U(u)du \qquad (5)$$

In evaluating the performance of the s-chart we have:

$$P_k(E \mid \hat{\sigma}) = 1 + F_W[(n-1)B^2_{\alpha/2;n}u^2/k^2] - F_W[(n-1)B^2_{1-\alpha/2;n}u^2/k^2]$$

where $F_W(\cdot)$ is the cumulative distribution function (CDF) of $W = (n-1)S^2/\sigma^2$, and u is the value of $U = \hat{\sigma}/\sigma$.

In evaluating the performance of the R-chart we have

$$P_k(E \mid \hat{\sigma}) = 1 + F_W[D_{\alpha/2;n}u/k] - F_W[D_{1-\alpha/2;n}u/k]$$

where $W = R/\sigma$.

Our simulation study indicates that in the event that the preliminary samples are contaminated, the $\bar{x}$, R and s charts constructed with robust probability limits outperform those constructed with the classical non-resistance scale estimator. To avoid lengthy discussions, we shall only report the performance of the s-chart with robust probability limits constructed from the Gaussian and contaminated Gaussian preliminary samples. As the sampling distribution of $U = \hat{\sigma}/\sigma$ cannot be obtained explicitly for the scale estimators considered in Section 3, the method of Monte Carlo simulation is used to evaluate the $ARL(k)$ from equation (5).

A three steps Monte Carlo method used in our simulation study of the s-chart is summarized as follow:

Step 1: Generate m preliminary samples from each of the $N(0,1)$, $CN(p,a)$, $C\chi^2(p,c)$ and $CSlash(p)$ distributions with presumed values of p, a and c.

Step 2: Estimate the process standard deviation σ using each of the six scale estimators considered in Section 3 from each set of m preliminary samples generated in Step 1, and subsequently evaluate the probability limits of the s-chart.

Step 3: Compute the $ARL(k)$ of the s-chart required to detect a shift of process standard deviation from σ to $k\sigma$ in future production process.

Previous studies based on the normality assumption [17, 18], and that under the non-normality assumption [5], reveal that the number of preliminary samples used in estimating the location and scale parameters and subsequently the control limits of the control charts should be much greater than the usually suggested number of 20 to 30, especially when the sample size is small. Consequently 200 samples are used in our simulation study to obtain a reliable estimate of the unknown scale parameter. The entries of Tables 5 and 6 are evaluated using the sample mean Monte Carlo integration technique with 100,000 replications of $m = 200$ preliminary samples of size n. The estimates of S_c are computed from (3) with $c = 7$. The estimates of S_{box} are computed from (4) with $k_l = k_u = 2.563$ ($k_l = k_u = 2.239$) which correspond to the probability of $\alpha_0 = 0.05$ that one or more observations in a Gaussian sample of size $n = 10$ ($n = 20$) will be wrongly classified as outliers [10].

Table 5. The $ARL(k)$ values of the s-chart with robust and resistant probability limits constructed based on 200 preliminary samples, each of size n taken from the $CSlash(p)$ distribution. Note that p=0 corresponds to uncontaminated $N(0,1)$ preliminary samples. The $ARL(k)$ values required to detect shifts of process standard deviation from σ to $k\sigma$, k=1.0(.2)2.0, are computed from equation (4) by generating 100,000 replications of m=200 preliminary samples. The underlined values are the smallest $ARL(k>1)$ values amongst the estimators considered

p	SCALE ESTIMATOR	$N(0, k^2)$, n=10						$N(0, k^2)$, n=20					
		k=1.0	k=1.2	k=1.4	k=1.6	k=1.8	k=2.0	k=1.0	k=1.2	k=1.4	k=1.6	k=1.8	k=2.0
0.00	sbar/c_4	363.2368	37.8087	7.8538	3.2935	2.0137	1.5162	362.7415	18.5699	3.4644	1.6517	1.2113	1.0724
	Q	359.4165	38.3286	7.9040	3.3035	2.0169	1.5175	359.1402	18.6770	3.4701	1.6526	1.2116	1.0725
	S_c	361.5276	38.0216	7.8739	3.2975	2.0149	1.5167	361.1422	18.5927	3.4647	1.6517	1.2113	1.0724
	S_{box}	361.4561	38.0432	7.8759	3.2978	2.0150	1.5167	358.9010	18.6967	3.4714	1.6529	1.2116	1.0725
	S_n	356.9569	38.7020	7.9402	3.3109	2.0192	1.5185	356.5147	18.7943	3.4775	1.6540	1.2120	1.0727
	MAD	354.4469	39.1519	7.9848	3.3200	2.0222	1.5198	352.0272	19.0568	3.4953	1.6573	1.2130	1.0731
0.02	sbar/c_4	205.2956	199.0878	86.2720	43.3979	26.0717	17.8867	61.3026	167.4954	91.7366	47.1083	27.8417	18.6452
	Q	390.4491	49.8907	9.3731	3.6850	2.1643	1.5885	378.1946	24.7911	4.0295	1.7798	1.2536	1.0882
	S_c	380.1821	46.1023	9.4086	3.8594	2.2984	1.6815	377.2361	21.7839	3.7700	1.7214	1.2343	1.0810
	S_{box}	383.0739	44.5854	8.7227	3.5205	2.1018	1.5587	372.2335	21.2184	3.7094	1.7078	1.2299	1.0793
	S_n	383.3280	47.9023	9.1135	3.6163	2.1375	1.5755	374.8779	23.6769	3.9272	1.7565	1.2459	1.0854
	MAD	376.8555	46.6835	8.9469	3.5707	2.1194	1.5667	368.3157	22.9797	3.8566	1.7398	1.2403	1.0833
0.05	sbar/c_4	47.3016	145.5351	174.8892	131.9559	91.1070	63.7707	4.3395	26.5947	85.9969	108.3880	93.1436	71.4968
	Q	393.4073	76.2231	12.4343	4.4247	2.4386	1.7176	337.3242	39.4657	5.1848	2.0240	1.3327	1.1181
	S_c	392.4145	61.6531	12.1192	4.8480	2.8083	2.0098	375.4328	28.2195	4.3397	1.8471	1.2752	1.0964
	S_{box}	399.7634	58.0074	10.3789	3.9365	2.2606	1.6373	376.7910	26.0989	4.1405	1.8041	1.2616	1.0913
	S_n	395.4308	67.3680	11.4212	4.1835	2.3500	1.6761	354.3626	34.5000	4.8088	1.9461	1.3076	1.1086
	MAD	392.7213	62.0328	10.7907	4.0289	2.2921	1.6488	361.9599	31.1213	4.5357	1.8876	1.2886	1.1015
0.10	sbar/c_4	6.8023	21.4686	59.3729	109.8867	136.4888	134.8130	1.0870	1.5751	3.8837	13.2797	35.2312	61.3120
	Q	314.8800	158.4616	21.3347	6.3105	3.0847	2.0085	206.0849	94.5852	8.5847	2.6461	1.5240	1.1905
	S_c	369.0997	100.3950	18.8346	7.0187	3.7951	2.5640	313.1811	46.1713	5.6869	2.1311	1.3681	1.1350
	S_{box}	376.7516	95.6481	14.7977	5.0189	2.5681	1.8265	340.1191	38.6886	5.1263	2.0119	1.3288	1.1166
	S_n	349.4499	122.3037	17.4177	5.5124	2.8181	1.8902	250.1110	69.9104	7.1576	2.3974	1.4491	1.1622
	MAD	368.7002	102.4379	15.2769	5.0522	2.6590	1.8183	290.2133	54.5494	6.1899	2.2176	1.3934	1.1411

Table 6. The *ARL(k)* values of *s*-chart with resistant probability limits constructed based on 200 preliminary samples, each of size n=20, taken from the $CN(p, a)$ or $C\chi^2(p, c)$ distribution with p=0.05. The *ARL(k)* values required to detect shifts of process standard deviation from σ to $k\sigma$, k=1.0(.2)2.0, are computed from equation (4) by generating 100,000 replications of m=200 preliminary samples. The underlined values are the smallest $ARL(k>1)$ values amongst the estimators considered

a	SCALE ESTIMATOR	$CN(p=0.05, a)$ $N(0, k^2)$ k=1.0	k=1.2	k=1.4	k=1.6	k=1.8	k=2.0	c	$C\chi^2(p=0.05, c)$ $N(0, k^2)$ k=1.0	k=1.2	k=1.4	k=1.6	k=1.8	k=2.0
1.0	sbar/c_4	362.6897	**18.5556**	**3.4630**	**1.6515**	**1.2112**	**1.0724**	0.0	362.6897	**18.5556**	**3.4630**	**1.6515**	**1.2112**	**1.0724**
	Q	360.0271	18.7984	3.4818	1.6554	1.2125	1.0728		360.0271	18.7984	3.4818	1.6554	1.2125	1.0728
	S_c	360.9516	18.5739	3.4629	1.6512	1.2111	1.0723		360.9516	18.5739	3.4629	1.6512	1.2111	1.0723
	S_{box}	358.3636	18.6755	3.4691	1.6523	1.2115	1.0725		358.3636	18.6755	3.4691	1.6523	1.2115	1.0725
	S_n	352.1063	18.2543	3.4248	1.6416	1.2079	1.0711		352.1063	18.2543	3.4248	1.6416	1.2079	1.0711
	MAD	351.8577	19.0521	3.4946	1.6571	1.2130	1.0731		351.8577	19.0521	3.4946	1.6571	1.2130	1.0731
2.0	sbar/c_4	286.1798	54.3781	6.2119	2.2257	1.3964	1.1422	2.0	86.5691	301.9073	25.4487	4.8198	2.0955	1.3985
	Q	352.7652	35.2110	4.8713	1.9600	1.3122	1.1104		346.3140	36.9910	5.0038	1.9872	1.3209	1.1137
	S_c	364.8808	31.9087	4.6209	1.9081	1.2955	1.1040		376.1107	27.8883	4.2957	1.8385	1.2729	1.0955
	S_{box}	370.2440	**29.6031**	**4.4310**	**1.8671**	**1.2822**	**1.0990**		376.9078	**25.9013**	**4.1237**	**1.8004**	**1.2604**	**1.0908**
	S_n	364.4395	31.2965	4.5618	1.8945	1.2910	1.1023		363.5330	31.5613	4.5818	1.8986	1.2923	1.1028
	MAD	363.8640	30.4040	4.4798	1.8759	1.2848	1.1000		364.8950	29.8692	4.4365	1.8666	1.2818	1.0989
3.0	sbar/c_4	106.2112	248.7849	17.5444	3.9387	1.8820	1.3231	3.0	23.3427	193.4881	203.1537	23.3687	5.2032	2.3233
	Q	293.9124	52.2071	6.0621	2.1962	1.3871	1.1387		318.8216	44.6344	5.5508	2.0970	1.3559	1.1269
	S_c	344.4789	37.5319	5.0481	1.9967	1.3240	1.1148		373.3660	29.0194	4.3878	1.8583	1.2793	1.0979
	S_{box}	361.4250	**32.5444**	**4.6631**	**1.9162**	**1.2981**	**1.1050**		376.2624	**26.4923**	**4.1736**	**1.8114**	**1.2640**	**1.0921**
	S_n	328.9562	41.8488	5.3481	2.0559	1.3428	1.1219		348.5014	36.2257	4.9390	1.9730	1.3163	1.1119
	MAD	343.7064	37.4180	5.0151	1.9872	1.3207	1.1136		356.8591	33.0864	4.6894	1.9200	1.2991	1.1054

Table 5 reveals that when the preliminary samples are uncontaminated (corresponds to $p=0$), then as expected the probability limits of the s-chart constructed with the classical scale estimator $\bar{s}/c_4$ yields in-control $ARL(k=1)$ value closest to the target value 370.37 and has the smallest out-of-control $ARL(k>1)$ values. However, when the preliminary samples data are contaminated with the long-tailed $CSlash(p)$ distribution with $p=0.02$, 0.05, 0.10, the s-chart constructed with the estimator $\bar{s}/c_4$ not only leads to out-of-control $ARL(k>1)$ values larger than that constructed with the robust estimators but its in-control $ARL(k=1)$ values are also unacceptably small. For instance, when preliminary samples of size $n=20$ are taken from the $CSlash(p)$ distribution with $p=0.10$, the s-chart constructed with the estimator $\bar{s}/c_4$ yields in-control $ARL(k=1)$ value of 1.087 which is equivalent to a high false alarm rate of 92%. This is due to the fact that the sampling distribution of the sample standard deviation S is highly skewed to the right, thus even a small to moderate over-estimate of σ would result in its lower probability limit cuts off more than the intended 0.135% area in the fat left tail of the distribution of S and consequently leads to higher false alarm rate. In contrast, a s-chart constructed with the robust scale estimators not only leads to smaller $ARL(k>1)$ values but also has the required large $ARL(k=1)$ values.

Table 5 also clearly reveals that the probability limits of the s-chart constructed using the robust estimator S_{box} yields the best $ARL(k=1)$ and the smallest $ARL(k>1)$ values amongst the estimators considered. For example, when preliminary samples of size $n=20$ are taken from the $CSlash(p)$ distribution with $p=0.10$, the s-chart constructed with the estimator S_{box} not only yields the smallest out-of-control $ARL(k>1)$ values but also provide the smallest false alarm rate as it has the largest in-control $ARL(k=1)$ value of 340.12 amongst the estimators considered.

Examination of Table 6 reveals that the s-chart constructed with S_{box} yields the smallest out-of-control $ARL(k>1)$ values when preliminary samples of size $n=20$ are taken from the contaminated $CN(p,a)$ and $C\chi^2(p,c)$ distributions with $p=0.05$. Note that $a=1$ and $c=0$ correspond to the case when the preliminary samples are uncontaminated and are taken from the standard Gaussian distribution.

REMARKS

The proposed control chart with robust probability limits is constructed by assuming that the underlying distribution of the quality characteristic is known and its scale parameter is estimated using resistant estimators. Our simulation study indicates that $\bar{x}$, R and s charts constructed with robust probability limits outperform the classical Shewhart chart when the preliminary samples are drawn from a contaminated Gaussian distribution. However, to avoid lengthy discussions, we only report the performance of the s-chart constructed with robust probability limits. The s-chart constructed with its probability limits evaluated using the estimator S_{box} yields the best in-control ARL and the smallest out-of-control ARL values amongst the robust and resistant estimators considered. Our results obtained from this study also reveal that the proposed estimator S_{box} is the most efficient and resistant estimator of scale amongst the estimators and contaminated distributions considered.

It cannot be denied that the robust estimation of the location and scale parameters is computation intensive and therefore clearly excludes the use of a hand-held calculator. However, this should not be an issue at this modern age where computing facilities are readily available and accessible at low cost.

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