Hilbert-space stability analysis

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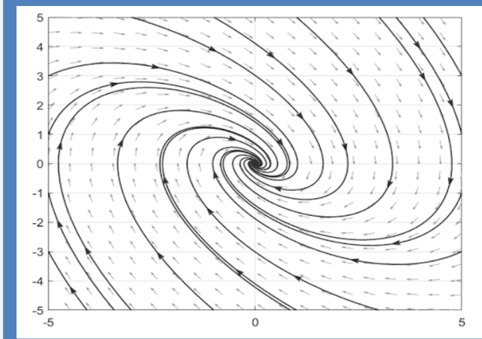
Classical stability analysis

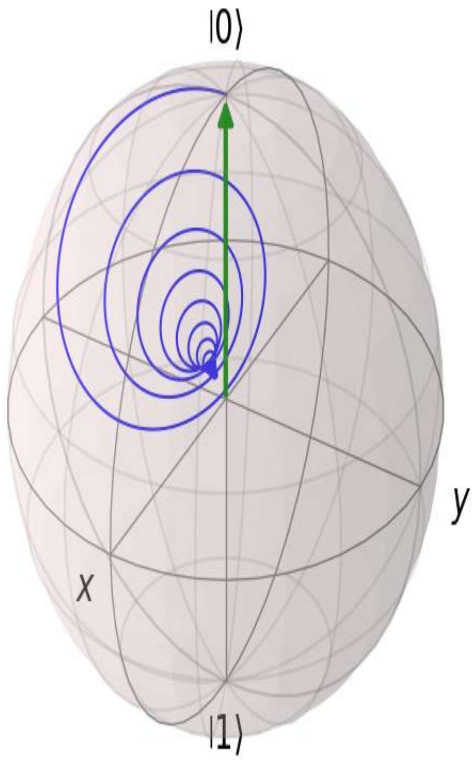
Control systems in engineering are generally nonlinear, making stability analysis complex. Nonlinear system stability analysis determines whether a system returns to an equilibrium point after a small perturbation; thus, the analysis is performed around these points. The key methods used in the field are Lyapunov stability and the linearization approach.

Hilbert space analysis:

Hilbert space is an abstract space where quantum states live. It is generally defined as a complex vector space. Moreover, Hilbert space offers tools for comparing quantum states, including inner products, fidelity, and state-space distance. This suggests that stability could be studied directly through the evolution of quantum-state trajectories.



A stable phase plot (or phase portrait) shows paths of a system moving inward toward a steady rest point, known as a **stable equilibrium.**



Qubit representation

Beyond phase plane: stability of nonlinear system in Hilbert space

Beyond the Phase Plane: Stability of Nonlinear Systems in Hilbert Space explores the possibility of representing nonlinear system dynamics in Hilbert space and analyzing stability through quantum-state trajectories. Instead of examining convergence or divergence around equilibrium points only within the classical state space, the system states can be mapped into quantum states in Hilbert space. The evolution of these states can then be analyzed using measures such as fidelity, inner products, and state-space distance to datermine how a small perturbation affects the system over time. In this representation, stability can be interpreted according to whether a perturbed quantum-state trajectory remains close to, converges towards, or diverges from a reference state. This perspective provides a potential bridge between classical nonlinear stability analysis and quantum-state representations, offering an alternative domain in which the behaviour of nonlinear dynamical systems may be investigated.

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